Answers to Central Limit Theorem Practice Problems in Notes

Graphical answers are not shown below. Graphical answers discussed and shown in class.

1a. X ~ N(10,2) so P(9.5 ≤ X ≤ 10.5) = normalcdf (9.5, 10.5, 10, 2) = 0.1974

1b. For n = 16 $\frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{16}} = 0.5$ so $\overline{X}$ ~ N(10,.5): P(9.5 ≤ $\overline{X}$ ≤ 10.5) = normalcdf (9.5, 10.5, 10, .5) = 0.6826

1c. For n = 25 $\frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{25}} = 0.4$ so $\overline{X}$ ~ N(10,. 4): P(9.5 ≤ $\overline{X}$ ≤ 10.5) = normalcdf (9.5, 10.5, 10, . 4) = 0.7887

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2. a. As n gets larger in the denominator, the quotient $\frac{\sigma}{\sqrt{n}}$ gets smaller. This means that as the sample size gets larger, the standard deviation of the sample average $\overline{X}$ is getting smaller and smaller.

b. Thisaffects graph of the distribution for $\overline{X}$. As the sample size n gets larger, the graph becomes less spread out and more concentrated about the mean.

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3. μ = σ = 5 m = 1/5 = 0.2 X ~ Exp (0.2) $\overline{X}$ ~ N(μ , $\frac{\sigma}{\sqrt{n}}$) $\overline{X}$ ~ N(5 , $\frac{5}{\sqrt{n}}$)

a. P(4 ≤ X ≤ 6) = $e^{(-.2*4)} - e^{(-.2*6)}$ = 0.1481

b. For n = 36 $\frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{36}} = 0.8333$ so $\overline{X}$ ~ N(5,.8333): P(4 ≤ $\overline{X}$ ≤ 6) = normalcdf (4, 6, 5, .8333) = 0.7699

c. For n = 64 $\frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{64}} = 0.625$ so $\overline{X}$ ~ N(5,.625): P(4 ≤ $\overline{X}$ ≤ 6) = normalcdf (4, 6, 5, .625) = 0. 8904

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4. X~U(6, 16) h = 1/(16–6) = 1/10 = 0.1

μ = (6 + 16)/2 = 11 σ = (16 – 6)/ $\sqrt{12}$ = 10/$\sqrt{12}$ ≈ 2.89 $\overline{X}$ ~ N(11 , $\frac{2.89}{\sqrt{n}}$)

a. P(4 ≤ X ≤ 6) = (12 – 10)(0.1) = 0.2

b. For n = 36 $\frac{\sigma}{\sqrt{n}} = \frac{2.89}{\sqrt{36}} \approx 0.482$ so $\overline{X}$ ~ N(11,.482): P(10 ≤ $\overline{X}$ ≤ 12) = normalcdf (10, 12, 11, .482) = 0.9620

c. For n = 64 $\frac{\sigma}{\sqrt{n}} = \frac{2.89}{\sqrt{64}} \approx 0.361$ so $\overline{X}$ ~ N(11,.361): P(10 ≤ $\overline{X}$ ≤ 12) = normalcdf (10, 12, 11, .361) = 0.9944

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5. The shape of the distribution of the sample averages is the shape of a normal distribution (mound shaped and symmetric) even when the shape of the original distribution of individuals is not. The shape of the distribution of sample averages does not follow the shape of the original distribution of individuals.

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6. μ = 14.1 σ = 2.5

a. We don't know what the distribution is for temperature of released water. Even though we know the mean and standard deviation, we only know that it is skewed right. The exact distribution is unknown. We need to know the distribution in order to calculate a probability for the temperature of released water on any one particular (individual) day.

b. However, we can find the probability for averages, because averages follow a normal distribution according to the CLT.

For n = 42 days, $\frac{\sigma}{\sqrt{n}} = \frac{2.5}{\sqrt{42}} \approx 0.386$ so $\overline{X}$ ~ N(14.1 , 0.386):

P($\overline{X}$ > 15) = normalcdf (15, 10^99, 14.1 , 0.386) = 0.0099

c. For samples of 60 days, the shape of the probability distribution for sample averages would NOT be skewed to the right, like the original distribution of temperatures on individual days. This is because the sample averages do not follow the same shape as the original distribution, but instead follow a normal distribution which is symmetric and is not skewed.

A. Question 1 revisited:

a. X ~ N(10,2) invnorm (.80, 10, 2) = 11.68 inches
80% of adult fish are less than 11.68 inches long

b. For n = 16 $\frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{16}} = 0.5$ so $\overline{X}$ ~ N(10,.5): invnorm (.80, 10, .5) = 10.42 inches
80% of all samples of 16 fish will have an average length less than 10.42 inches.

d. For n = 64 $\frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{64}} = 0.25$ so $\overline{X}$ ~ N(10,. 25): invnorm (.80, 10, . 25) = 10.21 inches
80% of all samples of 64 fish will have an average length less than 10.21 inches.

B. Question 4 revisited:

X~U(6, 16) h = 1/(16–6) = 1/10 = 0.1

μ = (6 + 16)/2 = 11 and σ = (16 – 6)/ $\sqrt{12}$ = 10/$\sqrt{12}$ ≈ 2.89 Therefore $\overline{X}$ ~ N(11 , $\frac{2.89}{\sqrt{n}}$)

a. X~U(6, 16) h = 1/(16–6) = 1/10 = 0.1 (k – 6) (0.1) = .75 Solve for k = 13.5 years
75% of students who ride the school busses are less than 13.5 years old

b. For n = 36 $\frac{\sigma}{\sqrt{n}} = \frac{2.89}{\sqrt{36}} \approx 0.482$ so $\overline{X}$ ~ N(11,.482): invnorm (.75, 11, .482) = 11.3
75% of all samples of 36 students riding school busses will have an average age less than 11.3 years