Chapter 4 : General Discrete Probability Distributions

Example D1: Students who live in the dormitories at a certain four year college must buy a meal plan. They must select from four meal plans: 10 meals, 14 meals, 18 meals, or 21 meals per week. The Food and Housing Office has determined that the 15% of students purchase 10 meal plan, 45% of students purchase the 14 meal plan, 30% of students purchase the 18 meal plan and 10% of students purchase the 21 meal plan.

a. What is the random variable? X = _____

_____

Notation: P(Event) = probability value

P(X = 10) is the probability that a student purchases a meal plan with 10 meals per week

P(X >14) is the probability that a student purchases a meal plan with more than 14 meals per week

b. Make a table that shows the probability distribution

This table is called the PDF

Probability Distribution Function

x =Number of Meals	Probability P(x)
10	
14	
18	
21	

We can create an extra column next to the PDF table to help calculate the mean

xP(x)

c. Find the probability that a student purchases more than 14 meals:

d. Find the probability that a student does not purchase 21 meals.

e. On average, how many meals does a student purchase per week in their meal plan?

Calculate the mean. **Mean = Expected Value: $\mu = \Sigma xP(x)$** μ = _____

f. Write a sentence that interprets the mean in the context of the problem.

NOTE that it is acceptable that the mean is not whole number; it can have a fraction or a decimal.

Example D2: A men's soccer team plays soccer 0, 1, or 2 days a week.
X = number of days the team plays soccer in a week

Illowsky, B., & Dean, S. Collaborative Statistics . Connexions, Dec. 5, 2008. http://cnx.org/content/col10522/1.29/

X	P(X)	XP(X)			
0	**0.2**				
1	**0.5**				
2	**0.3**				

Find the expected value and write a sentence that interprets in meaning in the context of the problem

Find the standard deviation: $\sigma = \sqrt{\sum (x-\mu)^2 P(x)}$

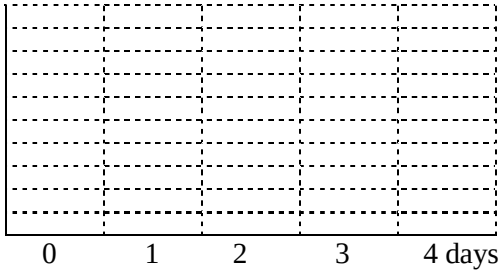
Example D3: **Using μ and σ to compare probability distributions**

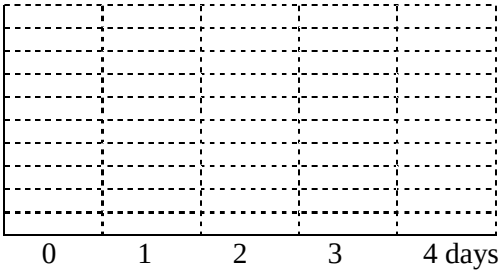
A men's soccer team plays soccer 0, 1, or 2 days a week.
A rival men's soccer team (Team B) plays soccer 0, 1, or 3 days a week.
The PDFs for both teams are shown below. X = number of days the team plays soccer in a week

Team A	
X	P(X)
0	0.2
1	0.5
2	0.3

Team B	
X	P(X)
0	0.2
1	0.7
2	0
3	0.1

a. Draw the relative frequency histograms of these probability distributions:





b. Which team plays soccer more days each week on average?

c. Which team shows more variability in their schedule?

Example D4: At the county fair, a booth has a coin flipping game. You pay $2.50 to flip **two fair coins**. If the result contains one or two heads, you win $3. If the result is two tails then there is no prize.
(a) Write the PDF for the amount won or lost in one game.
(b) Find the expected value for this game (Expected NET GAIN OR LOSS)
(c) Find the expected total net gain or loss if you play this game 50 times.

Example D5: Suppose you play a different game. In this game, you flip a **biased coin** twice.
Suppose that for this coin, P(HEAD) = 2/3 and P(TAIL) = 1/3.
In this game you do not pay in order to play.
You toss the coin twice, and then win or lose according to the following:
win $5 if you toss two tails; pay $2 if you toss two heads; pay $1 if you toss one head and one tail.
(a) Write the PDF for the amount won or lost in one game and (b) find the expected net gain or loss per game.

Example D5: In this game we roll ONE EIGHT SIDED DIE once.
(The eight sides of the die are numbered 1, 2, 3, 4, 5, 6, 7, 8)
Suppose that you win $6 if you roll an 8, win $2.50 if you roll a 2,
lose $2 if you roll an odd number, and
if you roll a 4 or 6 you neither win anything nor lose anything.

X = the monetary outcome for one game
(a) List the values that X can take on:

(b) Write the PDF for the amount won or lost in one game.
(c) Find the expected net gain or loss per game.
(d) Find the expected total net gain or loss if you play this game 80 times.

Example D6*: From Section 4.3 of Collaborative Statistics, Illowsky and Dean http://cnx.org/content/m16828/latest/*

Suppose you play a game of chance in which you choose 5 numbers from 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
You may choose a number more than once. You pay $2 to play and could profit $100,000 if you match all 5 numbers in order (you get your $2 back plus $100,000).
To win, you must get all 5 numbers correct, in order. Otherwise you lose the $2 that you paid to play.
Over the long term, what is your *expected* profit of playing the game?

Let X = the amount of money you profit.

Chapter 4 : Binomial Probability Distribution

A probability experiment has

- a fixed number **n** of repeated trials
- each trial has outcomes that we can classify as "success or "failure"
- outcome of trials are independent (*Outcome of a trial does not influence outcome of future trials*)

The probability of success on a single trial, **p**, is constant (the same) for all trials

We are interested in the number of successes, x, in n trials **Notation: X~B(n,p)**

Example B1: A college claims that 70% of students receive financial aid. Suppose that 4 students at the college are randomly selected. We are interested in the number of students in the sample who receive financial aid.

X = _____

p = the probability that a student receives financial aid: p = _____ q = 1−p = _____

X ~ B(4, 0.7) : Binomial with n = 4 and p = 0.7

X	P(x)
0	
1	
2	
3	
4	

Ways to get x successes in n trials

n = 4 **x = 1**	**n = 4** **x = 2**	**n = 4** **x = 3**
Abcd	**AB**cd	a**BCD**
a**B**cd	**A**b**C**d	**A**b**CD**
ab**C**d	**A**bc**D**	**AB**c**D**
abc**D**	a**BC**d	**ABC**d
	a**B**c**D**	
	ab**CD**	

a. Find the probability that AT MOST 2 of the students in the sample receive financial aid:

b. Find the probability that AT LEAST 3 of the students in the sample receive financial aid:

c. Find the **mean and the standard deviation** using the **shortcut formulas for the binomial distribution**:

$\mu = np$; $\sigma = \sqrt{npq}$ where $q = 1 - p$ **only for Binomial distribution.**

These shortcut formulas for μ and σ give the same results as the definitions $\mu = \Sigma x P(x)$, $\sigma = \sqrt{\sum (x-\mu)^2 P(x)}$ *with a lot less work!*

Formulas for Binomial Distribution: X~B(*n*,*p*) $P(X = x) = {}_nC_x\, p^x\, (1-p)^{n-x}$

P(X = *x*) is the probability of obtaining *x* successes in *n* independent trial

$\mu = np$; $\sigma = \sqrt{npq}$ **where *q* = 1 − *p*** **only for binomial distribution.**

${}_nC_x$ represents the number of ways (patterns) in which it is possible to get *x* successes in *n* trials

$${}_nC_x = \binom{n}{x} = \frac{n!}{x!(n-x)!}$$ *Where n! = n(n−1)(n−2)(n−3). . .(3)(2)(1) for integers n > 0*

Example 4! = (4)(3)(2)(1) = 24 3! = (3)(2)(1) = 6 2! = (2)(1) = 6 0! = 1 by definition

$${}_4C_2 = \frac{4!}{2!(4-2)!} = \frac{4!}{(2!)(2!)} = \frac{(4)(3)(2)(1)}{(2)(1)(2)(1)} = 6$$

${}_nC_x$ using calculator MATH PROB nCr : *Example*: 4 MATH PROB nCr 2 Enter

Binomial Distribution Calculator Instructions:

pdf : P(X = value) probability distribution function
cdf : P(X ≤ value) cumulative distribution function

TI 83, 84 2nd Distr

TI-89 APPS; 1: FlashApps; highlight Stats/List Editor ENTER F5: Distr

P(X= x)	**binompdf (n,p,x)**
P(X ≤ x)	**binomcdf (n,p,x)**
P(X < x)	**binomcdf (n,p,x − 1)**
P(X > x)	**1 − binomcdf (n,p,x)**
P(X ≥ x)	**1 − binomcdf (n,p,x − 1)**

Example B2: Success is not necessarily "good"

A pediatrician is interested in how many of his patients with flu develop pneumonia as a complication. The doctor has 200 patients with flu this winter. It is estimated that 2.6% of children with flu develop pneumonia as a complication. **We are interested in the number of his patients with flu who develop pneumonia.**

A patient who develops pneumonia is a "success", even though it is a "bad" outcome

X = (*description*)_____

p = (*description*)_____

p = (*value*) _____ X ~ _____

a. Find the probability that 3 patients with flu develop pneumonia

b. Find the probability that at most (≤) 3 patients with flu develop pneumonia

c. Find the probability that less than 3 patients with flu develop pneumonia

d. Find the probability that the number of patients with flu who develop pneumonia exceeds (is more than) 3

e. Find the probability that at least (≥) 3 get develop pneumonia

f. How many patients would you expect to develop pneumonia

Example B3: Make sure that the probability of success matches the definition of a success

Suppose the pediatrician is interested in how many of his 200 patients with flu do NOT develop pneumonia as a complication. It is estimated that 2.6% of children with flu develop pneumonia as a complication.

A patient who does NOT develop pneumonia is a success.

X = (*description*)_____

p = (*description*)_____

p = (*value*) _____ X ~ _____

a. find the probability that more than 190 of his 200 patients with flu do NOT develop pneumonia

b. How many patients with flu would you expect to NOT develop pneumonia

Example B4: *Example 4-10, Illowsky, B., & Dean, S. Collaborative Statistics. http://cnx.org/content/col10522/1.29/* It has been stated that about 41% of adult workers have a high school diploma but do not pursue further education. If 20 adult workers are selected find the probability that :

a. **at most 12** have a high school diploma but do not pursue further education.

b. **more than 15** have a high school diploma but do not pursue further education.

Chapter 4 : GEOMETRIC PROBABILITY DISTRIBUTION

A probability experiment has **Notation: X~G(p)**

- repeated trials
- each trial has outcomes that we can classify as "success or "failure"
- outcome of trials are independent (*Outcome of a trial does not influence outcome of future trials*)

The probability of success on a single trial, **p**, is constant (the same) for all trials

We are interested in the number of trials, x, needed to obtain the FIRST success.

Example G1:
According to an article in Scientific American (June 2004), approximately 15% of people are left-handed. For a random sample of people, it is reasonable to assume that left-handedness is independent. Suppose that you randomly select people from the population and ask them whether they are left handed.

We are interested in the number of people you need to ask in order to find one who is left handed (In other words, we are interested in the number of people to get the first occurrence of left-handedness)

X = _____

p = _____

p = _____ q = 1– p = _____

X ~ _____ What values can X take on?_____

a. Find the probability that we will need to ask 1, 2, 3, 4 people in order to find one who is left handed.

P(X = 1) =

P(X = 2) =

P(X = 3) =

P(X = 4) =

Formula:

b. Find the probability that we will need to ask AT MOST 2 people in order to find one who is left handed.

c. Find the probability that we will need to ask AT LEAST 3 people in order to find one who is left handed.

d. Find the probability that we will need to ask 3 OR 4 people in order to find one who is left handed.

e. How many people do we expect to ask in order to find one who is left handed?

Formula for the Geometric Distribution : $P(X = r) = q^{r-1}p$

$P(X = r)$ gives the probability that the first success occurs on the r^{th} trial

Notation: X~G(p) $\mu = \frac{1}{p}$ $\sigma = \sqrt{\frac{1}{p}\left(\frac{1}{p} - 1\right)}$ only for the geometric distribution

We are interested in the number of trials, x, needed to obtain the FIRST success.

It is so easy to calculate geometric probabilities, that for simple problems it is easier to do it by hand. If you want to use the distribution function on your calculator, they are similar to the binomial, but note that only two items are input into the geometpdf or geometcdf functions.

Geometric Distribution Calculator Instructions:

pdf : P(X = value) probability distribution function
cdf : P(X ≤ value) cumulative distribution function

TI 83, 84 2nd Distr

TI-89 APPS; 1: FlashApps; highlight Stats/List Editor ENTER F5: Distr

P(X= x)	**geometpdf (p,x)**
P(X ≤ x)	**geometcdf (p,x)**
P(X < x)	**geometcdf (p,x − 1)**
P(X > x)	**1 − geometcdf (p,x)**
P(X ≥ x)	**1 − geometcdf (p,x − 1)**

Examples from Section 4.6 Illowsky, B., & Dean, S. Collaborative Statistics. Connexions, Dec. 5, 2008. http://cnx.org/content/col10522/1.29/

Example G2: **ADDITIONAL PRACTICE PROBLEM: TRY AT HOME (***Based on example 4-14 in text***)** Suppose that you are looking for a chemistry lab partner. The probability that someone agrees to be your lab partner is 0.55. Since you need a lab partner very soon, you ask other students until you find one who agrees to be your lab partner.

X = _____

p = _____

p = _____ q = 1– p = _____

X ~ _____

a. What is the probability that you need to ask 4 students in order to find a lab partner?

b. What is the probability that you will need to ask at most 3 students?

c. What is the probability that you will need to ask more than 3 students?