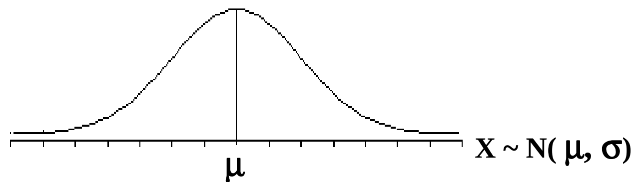
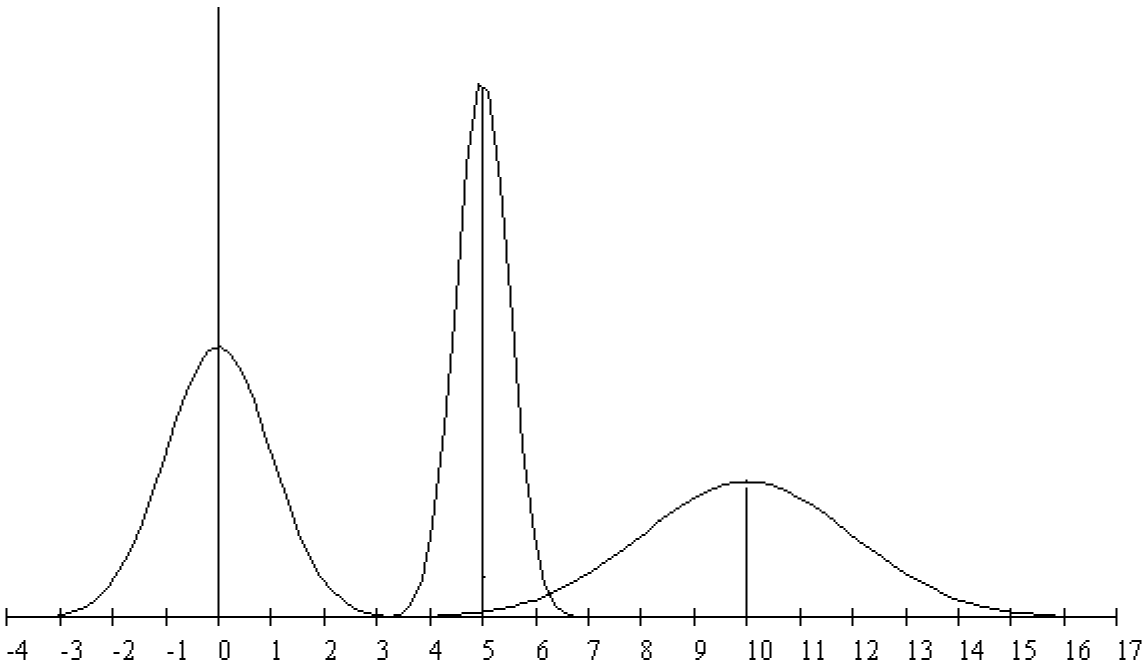
Exploring the Normal Probability Distribution





A. The normal distributions above have means of $\mu = 5$, $\mu = 10$ and $\mu = 0$. Identify graph has which mean.

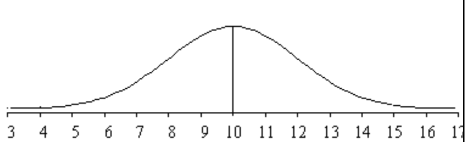
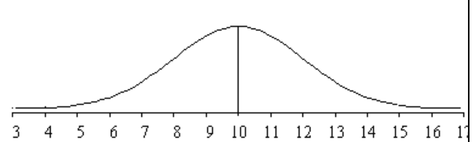
B. The normal distributions above have standard deviations of $\sigma = 0.5$, $\sigma = 1$and $\sigma = 2$.
Identify graph has which standard deviation.

C. Sketch a normal distribution with a mean of $\mu = -2$ and a standard deviation of $\sigma = 0.5$

<u>Shade</u> the area representing the probability.
<u>Label</u> the mean and values along axes and label the area.
Use your calculator to find the probabilities: **normalcdf (lower bound, upper bound, μ, σ)**

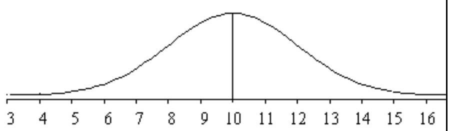
Write the answer as a probability statement *(equation with P(event) on the left and value of probability on the right).*

FINDING AREA IN BETWEEN: X ~ N(10, 2)

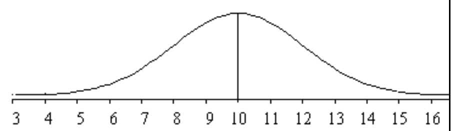
P(7 < X < 9)	**P(11 < X < 14)**
	

Use your calculator to find the probabilities: **normalcdf (lower bound, upper bound, μ, σ)**

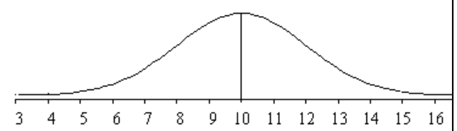
X ~ N(10, 2)



P(8 < X < 12) =

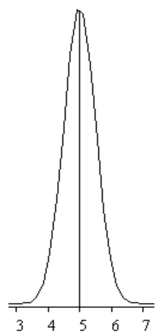


P(6 < X < 14) =

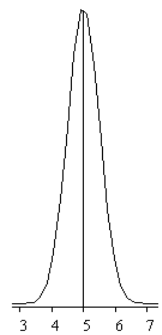


P(4 < X < 16) =

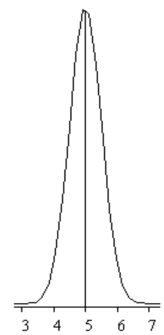
Y ~ N(5, 0. 5)



P**(4. 5 < Y < 5. 5) =**



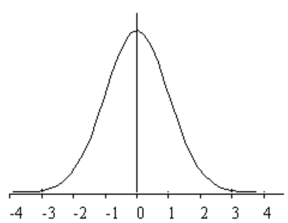
P(4 < Y < 6) =



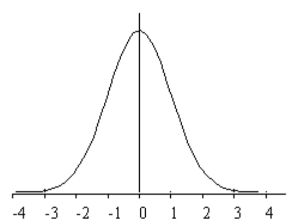
P(3. 5 < Y < 6. 5) =

Z ~ N(0,1)

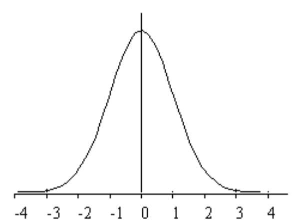
Standard Normal Distribution



P(−1 < Z < 1) =



P(−2 < Z < 2) =



P(−3 < Z < 3) =

For ANY Normal Probability Distribution, the probability for the interval

$\mu \pm 1\sigma$: within ± 1 standard deviations from the mean is ________

$\mu \pm 2\sigma$: within ± 2 standard deviations from the mean is ________

$\mu \pm 3\sigma$: within ± 3 standard deviations from the mean is ________

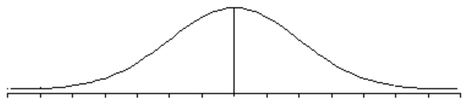
This result is commonly known as the Empirical Rule

Normal Probability and Inverse Normal using your Calculator

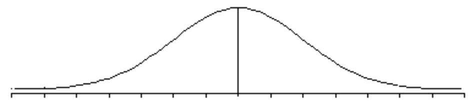
FINDING AREA TO THE RIGHT

X ~ N(10, 2)

normalcdf(lower, 1E99, μ, σ)
normalcdf(lower, 10^99, μ, σ)



P(X > 9) =



P(X ≥ 13) =

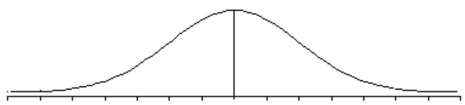
FINDING AREA TO THE LEFT

X ~ N(10, 2)

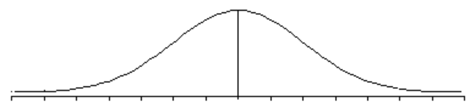
normalcdf((–) 1E99, upper, μ, σ)

normalcdf((–) 10^99, upper, μ, σ)

use (–) key for negative number



P(X ≤ 8) =

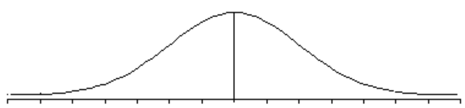


P(X < 11.5) =

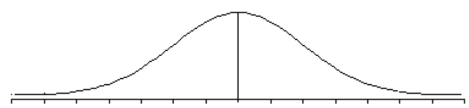
FINDING PERCENTILES: FINDING X VALUE WHEN WE KNOW THE AREA TO THE LEFT

X ~ N(10, 2)

invnorm (area to the left, μ, σ)



25th percentile



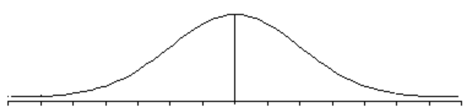
80th percentile

FINDING X VALUE WHEN WE KNOW THE AREA TO THE RIGHT

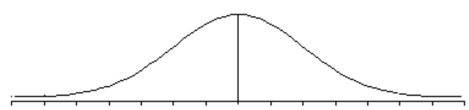
X ~ N(10, 2)

First find the area to the left
Then use the calculator
invnorm (area to the left, μ, σ)

Calculator must have area to the ***left*** *to use* invnorm



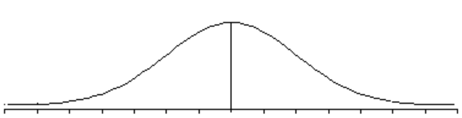
P(X ≥ k) = 0.60 Find k



P(X > k) = 0.30 Find k

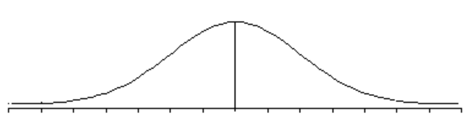
FINDING TWO X VALUES WHEN WE KNOW THE AREA IN THE MIDDLE

X ~ N(10,2)

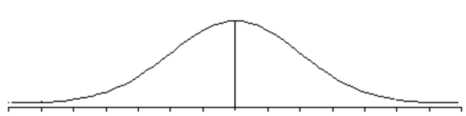


P(c ≤ X ≤ d) =0 .90
Find c and d:

1. First figure out the area to the left of the lower bound.



2. Then figure out area to the left of the upper bound



3. Then use the calculator with invnorm TWICE to find the lower and upper bounds:

lower bound =

$$invnorm\left(\begin{matrix}area\ to\ left\ of\\ lower\ bound\end{matrix}\quad ,\mu\quad ,\sigma\right)$$

upper bound =

$$invnorm\left(\begin{matrix}area\ to\ left\ of\\ upper\ bound\end{matrix}\quad ,\mu\quad ,\sigma\right)$$

<u>Exploring Connections between Percentiles and Z-Scores</u>

Two students, John and Ali, from different high schools wanted to determine who had the higher GPA when compared to his school. The GPAs at each high school follow a normal distribution.

Dean, S., & Illowsky, B. Collaborative Statistics, Chapter 2(10/27/2008) Connexions *http://cnx.org/content/m10522/*

Student	Student's GPA	School Mean GPA	School Standard Deviation
John	2.85	3.0	0.7
Ali	77	80	10

Which student's GPA would be considered better (John or Ali) when each is compared to the other students at his own school?

X = GPA of a student at John's school: X~________

Y = GPA of a student at Ali's school: Y~_________

Calculate and compare Z-scores (we did this in chapter 2; chapter 6 repeats this concept)

John	Ali

Compare percentiles for John and Ali by calculating probabilities using their actual GPAs

John: Find $P(X \leq 2.85)$ using X~N(3.0, 0.7)	Ali: Find $P(Y \leq 77)$ using Y~N(80, 10)

Compare percentiles for John and Ali by calculating probabilities using their z-scores

John: Find $P(Z < -0.2143)$ using Z ~ N(0, 1)	Ali: Find $P(Z < -0.30)$ using Z ~ N(0, 1)

<u>CONCLUSION:</u> **__________'s GPA of ____ would be considered better than ________'s GPA of ____ when each is compared to other students at his own school.**

NOTE

- Using z-scores and percentiles lead to the equivalent results and the same conclusion..
- Using the specified normal distribution X~N(3.0, 0.7) and Y~N(80, 10) give the same probabilities and percentiles as are obtained by using the Z score with the standard normal distribution, Z ~ N(0, 1)