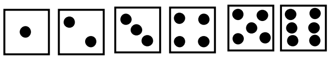
Probability: Events and Probabilities

PROBABILITY: long-run relative frequency; likelihood or chance that an outcome will happen.
A probability is a number between 0 and 1, inclusive,

Sample Space: Set of all possible simple outcomes

ROLLING 1 DIE:  **Sample Space:** **S = { 1 , 2 , 3 , 4 , 5 , 6 }**

Event: Combination of Outcomes; (also called "compound event")

AND, OR , NOT, GIVEN THAT relate two or more events to each other to create new compound events

Based on Example 3.2 Illowsky, B., & Dean, S. Collaborative Statistics. Connexions, http://cnx.org/content/col10522/1.29/

Event	A = {1, 3, 5}	B = {2, 4, 6}	C = { 3, 5}	D = {2, 4}	E = {1, 2, 3, 4}	G={4, 5, 6}
Probability	P(A) = $\frac{3}{6}$	P(B) =	P(C) = $\frac{2}{6}$	P(D) =	P(E) =	P(G) =

EXAMPLE 1: AND: A and B means BOTH events A and B occur:
The outcome that occurs satisfies both events A and B .
The outcome A and B includes items in common to both A and B: Intersection

Event A and E : { } Probability of Event A and E : P(A and E) =

Event D and G : { } Probability of Event D and G : P(D and G) =

OR: A or B means either event A occurs or event B occurs or both occur
Outcome that occurs satisfies event A or event B or both
The outcome A OR B is the union of items from these events.

Event A or E = { } Probability of Event A or E : P(A or E) =

Event D or G = { } Probability of Event D or G : P(D or G) =

NOT: COMPLEMENT A' means event A does NOT occur

A = {1, 3, 5} A'=

D = {2, 4} D'= $P(D')$ = $P(D) + P(D')$ =

COMPLEMENT RULE: For any event A: $P(A) + P(A') = 1$ $P(A') = 1 – P(A)$

EXAMPLE 2:

MUTUALLY EXCLUSIVE events are two events that can NOT both happen: P(A and B) = 0

List a pair of events above that are mutually exclusive and are NOT complements ___ , ___

List a pair of events above that are mutually exclusive and are complements: ___ , ___

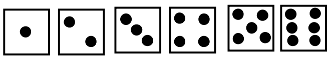
CONDITIONAL PROBABILITY: given that; if

Probability that event A occurs GIVEN THAT (if *we know that*) **B has occurred**

The outcome B that we know has occurred is called the condition

The condition B reduces the sample space to be smaller by eliminating outcomes that did not occur

Notation: P(A|B) P(Event| condition) the line **|** means: **given that** ; **if**

ROLLING 1 DIE:  **Sample Space: S = { 1 , 2 , 3 , 4 , 5 , 6 }**

Event	A = {1, 3, 5}	B = {2, 4, 6}	D = {2, 4}	E = {1, 2, 3, 4}	L={1, 2, 3}
Probability	P(A) = $\frac{3}{6}$	P(B) = $\frac{3}{6}$	P(D) = $\frac{2}{6}$	P(E) = $\frac{4}{6}$	P(L) = $\frac{3}{6}$

GIVEN THAT (if) we know that the outcome is even , the probability of rolling a 2 or 4 is 2/3.

The new "reduced" (smaller) sample space is { 2 , 4 , 6 } because we can exclude odd outcomes.

P(D|B) = P(2 or 4 | 2 or 4 or 6) = $\frac{2}{3}$ **{ ~~1~~ , (2) , ~~3~~ , (4) , ~~5~~ , 6 }**

EXAMPLE 3:

Find the probability rolling a number ≤ 3 *given that* (if) the outcome is even. P(L | B) =

Find the probability rolling a number ≤ 3 *given that* (if) the outcome is ≤ 4. P(L | E) =

Find the probability rolling a number ≤ 4 *given that* (if) the outcome is ≤ 3. P(E | L) =

Find the probability of rolling an odd number *given that* (if) the outcome was even. P(A | B) =

EXAMPLE 4: TOSSING A FAIR COIN TWICE:

A fair coin has equal probability of landing on Head (H) or Tail (T)

Sample space of outcomes for tossing a coin TWICE: **S =** { }

Find the probability of getting ONE HEAD in two tosses of the coin.

Find the probability of getting ONE HEAD in two tosses, *GIVEN THAT* AT LEAST ONE HEAD was obtained.

PRACTICE: EXAMPLE 5 : *DO AT HOME*

In a class, 50% of the students have long hair.

Of the females students, 90% have long hair.

12.5% of the male students have long hair.

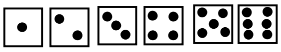
Events: L = long hair
F = female
M = male

Write each probability and the corresponding symbols for its event using proper notation**.**

INDEPENDENT EVENTS:
Two events are independent if and only if the probability of one event occurring is not affected by whether the other event occurs or does not occur.

- **Events A and B are INDEPENDENT if and only if P(A | B) = P(A)**
  This also means that P(B | A) = P(B)
- **Independence means that the "condition B " does not change the probability of event A**

EXAMPLE 6



ROLLING ONE DIE

Event	Even number	Number ≤ 4	Number ≤ 3
Event	B = {2, 4, 6}	E = {1, 2, 3, 4}	L={1, 2, 3}

The probability of rolling a number ≤ 4, given that the number is even: P(E | B) =

The probability of a number ≤ 4 : P(E) =

The probability of rolling a number ≤ 3, given that the number is even: P(L | B) =

The probability of a number ≤ 3 : P(L) =

EXAMPLE 7 Which of the following describe **independent events**?

Rolling two dice ________________________________

Selecting 2 cards consecutively from a deck of 52 cards, **without replacement _____________________**

Selecting 2 cards from a deck of cards, **with replacement** ________________

EXAMPLE 8: *Data from the FHDA district website for a past quarter:*
15% of students registered at De Anza College were Hispanic
15 % of students registered at Foothill College were Hispanic
15% of all students registered at FHDA District were Hispanic
H = event that a student was Hispanic D = event that a student attended De Anza
Were the events "student is Hispanic" and "student attends De Anza College" independent?
Show appropriate numerical justification for your answer.

EXAMPLE 9: http://www.censusindia.gov.in/2011-prov-results/indiaatglance.html
http://www.censusindia.gov.in/Census_Data_2001/India_at_glance/literates1.aspx
Adult (15+ years) literacy rates are 82.1% for men and 65.5% for women
The overall literacy rate is estimated as approximately 74%.
Note: The literacy rates have improved since 2001 when the rates were: Overall 64.8% Male: 75.3% Female: 53.7%

Consider the population of residents of India age 15 and over:
F = event that resident of India is female M = event that resident of India is male
L = event that a randomly selected resident of India is literate
Is the literacy rate in India independent of gender? *Justify your answer using appropriate probabilities.*

CONTINGENCY TABLES

A table displays data for two variables; the number of individuals or items in each category os shown. We can use the data in the table to find probabilities.

All probabilities <u>EXCEPT conditional probabilities</u> have the grand total in the denominator

<u>Conditional Probabilities</u>: The condition limits you to a particular row or column in the table. The **denominator will be the total for the row or column** in the table that corresponds to the condition

EXAMPLE 10: A large car dealership examined a sample of vehicles sold or leased in the past year. The vehicles were classified by type

- car, SUV, van, truck
- whether they were a sale of a new or used vehicle or whether the vehicle was leased.

	Car (C)	SUV (S)	Van (V)	Truck(T)	Total
New vehicle sale(N)	86	25	21	38	170
Used vehicle sale (U)	39	13	4	22	78
Vehicle Lease (L)	34	12	6	0	52
Total	159	50	31	60	300

Suppose a vehicle in the sample is randomly selected to review its sales or lease papers.

a. Find the probability that the vehicle was leased.

b. Find the probability that a vehicle is a truck.

c. Find the probability that a vehicle is NOT a truck.

d. Find the probability that the vehicle was a car AND was leased.

e. Find the probability that the vehicle was used GIVEN THAT it was a van.

f. Find the probability that the vehicle was a van GIVEN THAT it was used.

g. Find the probability that the vehicle was used AND was a van.

Addition Rule for OR Events: P(A or B) = P(A) + P(B) – P(A and B)

h. Find the probability that the vehicle was used OR was a van.

i. Find the probability that the vehicle was leased OR was a truck.

INDEPENDENCE in CONTINGENCY TABLES

An easy way to check if two events are independent in a contingency table is
Let the outcome represented by a column be the "condition"
Let the outcome represented by a row be the"event"

Compare : P(event in row) to P(event in row | condition in column)

If and only if these probabilities are equal, then the events are independent

EXAMPLE 11:

a. Are the events N and V independent?

	Car (C)	SUV (S)	Van (V)	Truck(T)	Total
New vehicle sale(N)	86	25	21	38	170
Used vehicle sale (U)	39	13	4	22	78
Vehicle Lease (L)	34	12	6	0	52
Total	159	50	31	60	300

Show your work to justify your answer using appropriate numerical evidence in the probabilities.

P(Event | Condition) = P(___ | ____) = P(Event) = P(___) =

Conclusion:_________________________ Reason ____________________

b. Are the events S and U independent?

	Car (C)	SUV (S)	Van (V)	Truck(T)	Total
New vehicle sale(N)	86	25	21	38	170
Used vehicle sale (U)	39	13	4	22	78
Vehicle Lease (L)	34	12	6	0	52
Total	159	50	31	60	300

Show your work to justify your answer using appropriate numerical evidence in the probabilities.

PRACTICE: EXAMPLE 12: INDEPENDENCE : *DO AT HOME*

From ch. 3, Collaborative Statistics Illowsky, B., & Dean, S. www.cnx.org

Solutions are in textbook chapter 3 online *or at end of chapter in the chapter pdf file*

Hiking Preference by Gender for a sample of 100 hikers	Near Coastline (C)	Near Lakes and Streams	Mountains	Total
Female (F)	18	16	11	45
Male	16	25	14	55
Total	34	41	25	100

Are the events "being female" and "preferring the coastline" independent?
Show your work to justify your answer using appropriate numerical evidence in the probabilities.

PROBABILITY RULES

Complement Rule: **$P(A') = 1 - P(A)$**

Conditional Probability Rule: **$P(A|B) = \dfrac{P(A \text{ AND } B)}{P(B)}$**

("given that")

Multiplication Rule for <u>AND</u> Events:

General Multiplication Rule: **$P(A \text{ and } B) = P(A \mid B)\, P(B)$**

IF AND ONLY IF events are INDEPENDENT: **$P(A \text{ and } B) = P(A)\, P(B)$**

Addition Rule for <u>OR</u> Events:

General Addition Rule: **$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$**

IF AND ONLY IF events are MUTUALLY EXCLUSIVE: **$P(A \text{ or } B) = P(A) + P(B)$**

EXAMPLE 13: ***Conditional Probability Rule***

At a large company:

34% of employees live over 30 miles away from the company's offices

52% of employees sometimes work from home.

28% of employees sometimes work from home AND live over 30 miles away from the company's offices

Let H = the event that the employee sometimes works at home.
Let F = be the event that the employee lives over 30 miles away from the company's offices

Find the probability that an employee sometimes works at home, **<u>given that</u>** the employee lives over 30 miles away from the company's offices

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PRACTICE: EXAMPLE 14: ***Conditional Probability Rule*** *DO AT HOME*

In a certain neighborhood, 65 % of residents subscribe to the Mercury News and 30% of residents subscribe to the SF Chronicle. These figures include the fact that 20% of residents subscribe to both.

M = event that a person subscribes to the Mercury News
C = event that a person subscribes to the Chronicle

Find the probability that a person subscribes to Mercury News **<u>given that</u>** the person subscribes to the SF Chronicle.

Complement Rule: **P(A′) = 1 – P(A)**

Addition Rule for <u>OR</u> Events:

General Addition Rule: **P(A or B) = P(A) + P(B) – P(A and B)**

IF AND ONLY IF events are MUTUALLY EXCLUSIVE: **P(A or B) = P(A) + P(B)**

EXAMPLE 15: ***Addition Rule for OR Events:***
In a certain neighborhood, 65% of residents subscribe to the Mercury News (event M)and 30% of residents subscribe to the SF Chronicle (event C). These figures include the fact that 20% of residents subscribe to both.

Find the probability that a person subscribes to Mercury News <u>OR</u> the SF Chronicle

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EXAMPLE 16: ***Addition Rules for OR Events***
a. When rolling two dice the probability of rolling a sum of 6 is 5/36 and the probability of rolling a double is 6/36.
The probability of rolling a 6 by rolling a 3 on each die is 1/36.

Find the probability of rolling a sum of 6 or a double.

Sample Space for sum when rolling 2 dice
<u>36 possible outcomes</u>

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

b. When rolling two dice the probability of rolling a sum of 3 is 2/36 and the probability of rolling a double is 6/36. **Find the probability of rolling a sum of 3 or a double.**

EXAMPLE 17: ***Addition Rules for OR Events (Harder)***

In a city, 50% of residents watch the 6PM news and 30% of residents watch the 11PM news.
35% of residents do not watch either the 6PM news or the 11PM news.

S = event that a person watches the 6 pm news ; E = event that a person watches the 11 pm news

(a) Find the probability that a person watches the 6PM news or the 11PM news *(or both)*.

(b) Find the probability that a person watches both the 6PM news and 11 PM news.

Conditional Probability Rule: $P(A|B) = \dfrac{P(A \text{ AND } B)}{P(B)}$
("given that")

Multiplication Rule for <u>AND</u> Events:

General Multiplication Rule (always true): **$P(A \text{ and } B) = P(A \mid B)\, P(B)$**

IF AND ONLY IF events are INDEPENDENT: **$P(A \text{ and } B) = P(A)\, P(B)$**

EXAMPLE 18: http://www.censusindia.gov.in/2011-prov-results/indiaatglance.html

India's population is 48.5% female and 51.5% male
Adult (15+ years) literacy rates are 82.1% for men and 65.5% for women
The overall adult literacy rate is estimated as approximately 74%.

Consider the population of residents of India age 15 and over. Assume that the population age 15 and over follows the same gender distribution: 48.5% female and 51.5% male

F = event that resident of India is female M = event that resident of India is male
L = event that a randomly selected resident of India is literate

a. Find the probability that a randomly selected person in India (age 15+) is both literate **<u>and</u>** female.

b. Find the probability that a randomly selected person in India (age 15+) is literate and male

c. Use the Conditional Probability Rule to find the probability that a randomly selected resident of India age 15 or over is female, **given that** the person is literate:

EXAMPLE 19: Consider the experiment of tossing a fair coin twice.
Find the probability of getting Head on both tosses.

Complement Rule: $P(A') = 1 - P(A)$

Conditional Probability Rule: $P(A|B) = \frac{P(A \text{ AND } B)}{P(B)}$
("given that")

Multiplication Rule for AND Events:

General Multiplication Rule: $P(A \text{ and } B) = P(A \mid B)\, P(B)$

IF AND ONLY IF events are INDEPENDENT: $P(A \text{ and } B) = P(A)\, P(B)$

Addition Rule for OR Events:

General Addition Rule: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

IF AND ONLY IF events are MUTUALLY EXCLUSIVE: $P(A \text{ or } B) = P(A) + P(B)$

PRACTICE EXAMPLE 20: *DO AT HOME*

In a class there are male and female students, and students with long or short hair
60% of the students in a class are female. 40% of the students are male. 50% of the students have long hair.45% of the students are female and have long hair. Of the male students, 12.5% have long hair.

Events: F = student is female M = student is male L = student has long hair

Identify each of these events and probabilities as stated in the words of the problem. Use correct symbols:

60% of the students in a class are female:___________ 40% of the students are male: ___________

50% of the students have long hair: ______________

45% of the students are female and have long hair: _____________________

Of the male students, 12.5% have long hair: _____________________

For parts a through e, show how to use an appropriate probability rule to calculate the probability. Write the probability statement using proper notation and show the work for each calculation.

a. Find the probability that a student has long hair if the student is female.

____________________________ Rule

b. Find the probability that a student is male and has long hair.

____________________________ Rule

c. Find the probability that a student is male or has long hair.

____________________________ Rule

d. Find the probability that a student is female and has short hair. *(requires a little creative thinking!)*

____________________________ Rule

e. What percent of the students with long hair are female?

____________________________ Rule

CHECKING IF TWO EVENTS ARE INDEPENDENT

- **Two events are independent if and only if P(A|B) = P(A)** This also means that P(B|A) = P(B)
- **IF AND ONLY IF events A and B are independent, P(A and B) = P(A) P(B)**
  You can use this equation to check for independence *IF you already know P(A and B)*

2 methods to check for independence

- **"Conditional" probability: Compare P(A|B) to P(A) .**
  If P(A|B)=P(A), the events are independent
- **"And" probability: Compare P(A and B) to P(A) P(B).**
  If P(A and B) = P(A) P(B), the events are independent

Do one method only. *Both methods always give the same result (if done correctly)*
Do whichever is easier with the information you know for the problem.

EXAMPLE 21: Check for Independence
A class is 60% female. 50 % of all students in the class have long hair.
45% of the students are female and have long hair.
75% of the female students have long hair.
Event F = student is female Event L = student has long hair
Are the events of being female and having long hair independent?

Check with Conditional Probabilities:

P(L | F) =_______ P(L) =_______ Does P(L | F) = P(L)? _______

Check using "AND" probabilities:

P(F and L) =_____ P(F) P(L) =____________________ Does P(F and L) = P(F) P(L) ? _______

Conclusion: ____

PRACTICE EXAMPLE 22: Test for Independence *DO AT HOME*
From Ch 3, Illowsky, B., & Dean, S. Collaborative Statistics. Connexions, 12/2008 http://cnx.org/content/col10522/1.29

60% of students take a math class and 50% of students take a science class.
30% of students take math class and a science class
Event G = student takes a math class Event H = student takes a science class
Are events G and H independent?

Test using "AND" probabilities:

P(G and H) =_____ P(G) P(H) =________________ Does P(G and H) = P(G) P(H) ? ____

Conclusion: ____

For this example, we are were not told any conditional probabilities. If we were to test with Conditional Probability, we have to use the conditional probability rule first to calculate it. For this problem it is easier to test for independence using the "and" probability.

TREE DIAGRAMS

Tree diagrams are a useful tool in solving probability problems
Each complete path through the tree represents a separate mutually exclusive outcome in the sample space.

1. Draw a tree representing the possible mutually exclusive outcomes
2. Assign conditional probabilities along the branches of the tree
3. Multiply probabilities along each complete path through the tree to find probabilities of each "AND" outcomes in the sample space.
4. Add probabilities for the appropriate paths of a tree to find the probability of a compound OR event.

EXAMPLE 23: *From Chapter 3 Section 3.7 Tree diagrams in Illowsky, B., & Dean, S. Collaborative Statistics. Connexions, Dec. 5, 2008. http://cnx.org/content/col10522/1.29*

An urn contains 11 marbles, 3 Red and 8 Blue. We are selecting 2 marbles randomly from the urn.
Draw the tree diagram. Show the events and probabilities for each branch and each complete path of the tree.

Select 2 marbles WITH REPLACEMENT:
Find the probability of selecting one marble of each color

Select 2 marbles WITHOUT REPLACEMENT
Find the probability of selecting one marble of each color

EXTRA PRACTICE PROBLEMS: USING PROBABILITY RULES
Do at home: solutions are posted on the Catalyst website

Practice Example A: Suppose H and J are events. **P(H) = 0.2 P(J) = 0.4 P(J and H) = 0.12**

(a) Are H and J independent?
(b) Find P(J | H)
(c) Find P(H | J)
(d) Find P(H or J)

Practice Example B: Suppose R and S are events**. P(R) =0.2 P(S) = 0.5 P(R | S) = 0.3**

(a) Are R and S independent?
(b) Find P(R and S)
(c) Find P(S | R)
(d) Find P(R or S)

Practice Example C: Suppose F and G are events. **P(F) = 0.4, P(G) = 0.3 P(F or G) = 0.6**

(a) Find P(G and F)
(b) Find P(G|F)
(c) Find P(F|G)
(d) Are F and G mutually exclusive?
(e) Are F and G independent?

Practice Example D: Redo example C using P(F or G) = 0.58 instead of 0.60

P(F) = 0.4, P(G) = 0.3 P(F or G) = 0.58

(a) Find P(G and F)
(b) Find P(G|F)
(c) Find P(F|G)
(d) Are F and G mutually exclusive?
(e) Are F and G independent?

EXTRA PRACTICE PROBLEMS FOR TREES

Do at home: solutions are posted on the Catalyst website

PRACTICE EXAMPLE E: TREE Source: http://www.censusindia.gov.in/2011-prov-results/indiaatglance.html

India's population is 48.5% female and 51.5% male
Adult (15+ years) literacy rates are 82.1% for men and 65.5% for women
Consider the population of residents of India age 15 and over. Assume that the population age 15 and over follows the same gender distribution: 48.5% female and 51.5% male

F = event that resident of India is female   M = event that resident of India is male
L = event that a randomly selected resident of India is literate

Draw a tree representing the possible mutually exclusive outcomes
Assign conditional probabilities along the branches of the tree
Multiply probabilities along each complete path through the tree and label the corresponding events
Use the tree to find the probability that an adult in India is literate

PRACTICE EXAMPLE F: TREE

Two statistics professors in Poland had their students do an experiment that showed that the Belgian one Euro coin is not a fair coin. It is a biased coin. In their experiment, it showed a 56% chance of landing on a "Head".

Suppose we do an experiment of tossing the Belgian one Euro coin twice in a row.

Draw a tree representing the possible mutually exclusive outcomes
Assign conditional probabilities along the branches of the tree
Multiply probabilities along each complete path through the tree and label the corresponding events
Use the tree to find the probability of getting at least one head.