NUMERICAL AND GRAPHICAL SUMMARIES OF QUANTITATIVE DATA: FREQUENCY DISTRIBUTIONS AND HISTOGRAMS

The frequency distribution table summarizes the data.
A HISTOGRAM is a bar graph displaying quantitative (numerical) data

Numerical data may be presented individually (ungrouped) or grouped into intervals

EXAMPLE 1: Individual Data Values *(ungrouped)* 2,5,3,1,2,4,1,2,3,1,1,2,7,4,2,3
Number of flowers on a plant, for a sample of 16 plants in a lab experiment

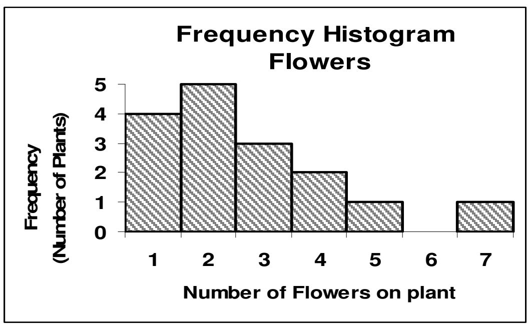
Number of Flowers	Frequency	Relative Frequency	Cumulative Relative Frequency
1			
2			
3			
4			
5			
6			
7			

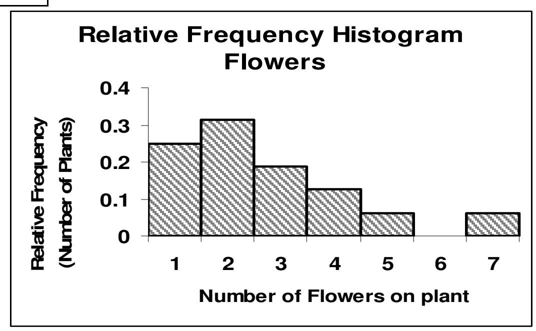
a. What percent of plants had 3 flowers?

b. What percent of plants had <u>at most</u> 3 flowers?

c. What percent of plants had <u>more than</u> 3 flowers?

d. What percent of plants had <u>at least 5</u> flowers?

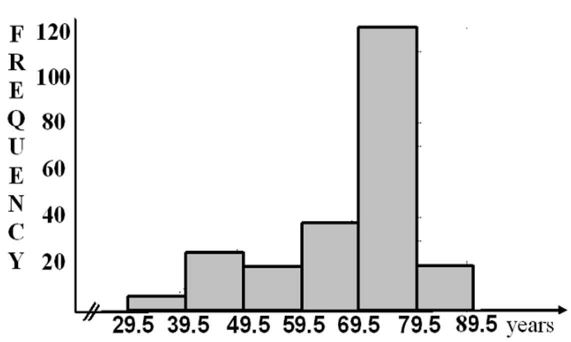




EXAMPLE 2: Life Expectancy at Birth In Years: *see page 3 for definitions of column titles*

2005 Life Expectancy data from U.S. Bureau of the Census International Data Base for 227 countries

Interval using Class Limits	Interval using Class Boundaries	Frequency	Relative Frequency	Cumulative Relative Frequency
30–39	29.5 to 39.5	6	6/227 = 0.026	0.026
40–49	39.5 to 49.5	25	25/227 = 0.110	0.137
50–59	49.5 to 59.5	19	19/227 = 0.084	0.220
60–69	59.5 to 69.5	38	38/227 = 0.167	0.388
70–-79	69.5 to 79.5	120	120/227 = 0.529	0.916
80–-89	79.5 to 89.5	19	18/227 = 0.084	1.000



Life Expectancy at Birth in Years for 227 countries

***Note: In Math 10*, we will use intervals of EQUAL WIDTH**

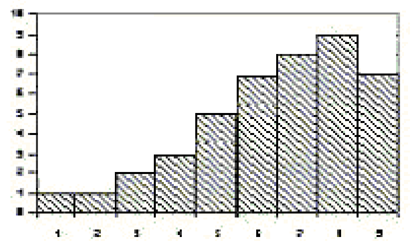
EXAMPLE 3: Ages of people:

Intervals varying in width:
0-5, 6-14, 15-19, 20-24, 25-29, 30-39, 40-49, 50-59, 60-64, 65-79, 80+

Intervals of EQUAL WIDTH:
0-9, 10-19, 20-29, 30-39, 40-49, . . . , 80-89, 90-99

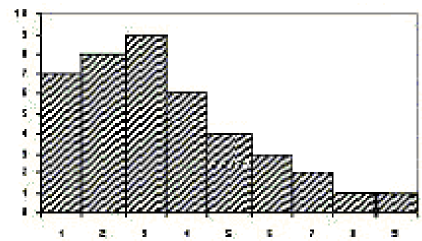
Shapes of Data Distributions

Skewed to the LEFT
(negatively skewed)



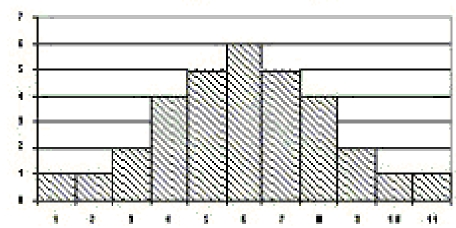
When data is skewed to the left,
generally the mean is less than the median

Skewed to the RIGHT
(positively skewed)



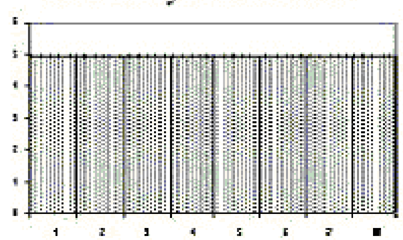
When data is skewed to the right,
generally the mean is greater than the median

Mound Shaped & Symmetric

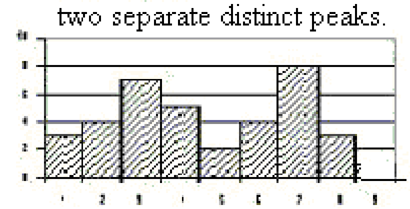


For symmetric data,
mean = median

Uniformly Distributed

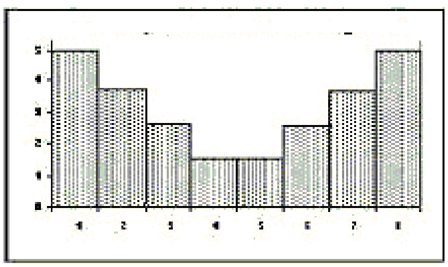


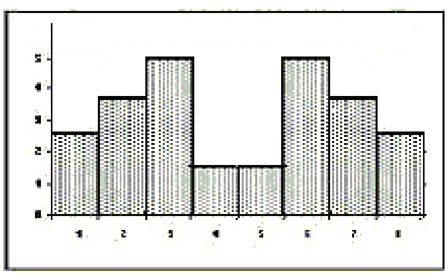
Bimodal
two separate distinct peaks.

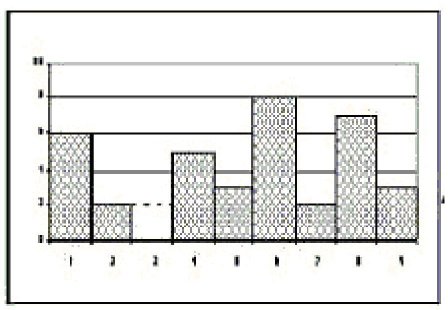


"hills" separated by a "valley"
Peaks do not need to be exactly the same height

Below are two additional examples
of symmetric data.
Note that these are not moundshaped:







If data do not fit one of these descriptive terms,
do not use a term that doesn't fit its shape.

Just describe what you see in the data if none
of these descriptive terms apply.

Definitions and Calculator Instructions

- **Class Limits: Lowest and highest possible data values in an interval.**
- **Class Boundaries: Numbers used to separate the classes, but without gaps.** Boundaries use one more decimal place than data and class limits. This prevents data values from falling on a boundary, so no ambiguity exists about where to place a particular data value
- **Class Width: Difference between two consecutive class boundaries**
  Can also calculate as difference between two consecutive lower class limits

  Age interval 30-39: 30 is the lower class limit 39 is the upper class limit
  Class boundaries are 29.5 to 39.5

  Age interval 40-49: 40 is the lower class limit 49 is the upper class limit
  Class boundaries are 39.5 to 49.5

  Class Width is 39.5 – 29.5 = 49.5 – 39.5 = 10

- **Class Midpoints: Midpoint of a class = (lower limit + upper limit) / 2**

  Age interval 30-39: class midpoint is (30 + 39)/2 = 34.5

- **Frequency = count = number of data values that lie in the interval**
  A **frequency distribution** counts the **number** of data items that fall into each interval.
- **Relative Frequency = proportion of data values that lie in the interval** $= \dfrac{\textbf{Frequency}}{\textbf{Number of Observations}}$
  A **relative frequency distribution** shows the **proportion (**fraction or percent) of data items in each interval.
- **Cumulative Relative Frequency**
  = sum of relative frequencies for all intervals up to and including current interval

Entering data into TI-83, 84 statistics list editor:

[STAT] "EDIT" Put data into list L1, press [ENTER] after each data value
If you have a frequencies for each value, enter frequencies into list L2, press [ENTER] after each value
[2^{nd}] [QUIT] to exit stat list editor after you have entered data, checked it and corrected errors.

HISTOGRAM instructions for the TI-83, 84: Assuming your data has been entered in list L1

[2nd] [STATPLOT] [1]

Highlight "**ON**" ; press **[ENTER]**
Type: Highlight histogram icon press **[ENTER]**
Xlist: **[2nd] [L1] [ENTER]**
Freq: If there is no frequency list and all data is in one list type **[1] [ENTER]**
OR If there is a frequency list, enter that list here **[2nd] [L2] [ENTER]**

Set the appropriate window and scale for the histogram

[WINDOW]
XMin: lower boundary of first interval **XMax**: upper boundary of last interval **Xscl** = interval width
Example: For intervals 10 to <20, 20 to <30, . . . 60 to <70: Xmin = 9.5 Xmax=69.5 Xscl=10
YMin = 0 Estimate **YMax** to be large enough to display the tallest bar
Select an appropriate value of **YScl** for the tick marks on the y-axis

[GRAPH] **Calculator constructs the histogram**

[TRACE] You can use the left and right cursors (arrow keys) to move from bar to bar.
The screen indicates the frequency (count, height) for the bar that the cursor is positioned on.

For TI-89 see the calculator instructions web handout

For TI-83, 84 Instructions for 1 variable statistics, see page 9 of notes.

EXAMPLE 4:
Student Total Headcount
Bay Area Community College Enrollment Fall 2009

25 Community Colleges comprising Regions III and IV of all community colleges (Bay and Interior Bay regions)

Note that the data has already been sorted into ascending numerical order.

Community College Campus	Enrollment
Marin	7106
Berkeley City	7288
Canada	7322
Alameda	7409
Merritt	7588
Contra Costa	8994
Napa	9488
Las Positas	9758
Skyline	10505
Evergreen Valley	10612
Los Medanos	10976
San Mateo	11273
Monterey	11806
Ohlone	12209
Solano	12261
San Jose City	12309
Mission	12680
Laney	14549
West Valley	14599
Chabot Hayward	15236
Foothill	19422
Diablo Valley	22123
Deanza	25265
San Francisco Ctrs	32354
San Francisco	33008

https://misweb.cccco.edu/mis/onlinestat/studdemo_coll_rpt_cube.cfm?RequestTimeout=1000
https://misweb.cccco.edu/mis/onlinestat/studdemo_coll_DL_cube.cfm?RequestTimeout=1000&College=100&YearTerm=0978&Colvar1=None&Colvar2=None&Colvar3=None

4a. Create a frequency/relative frequency/cumulative relative frequency table

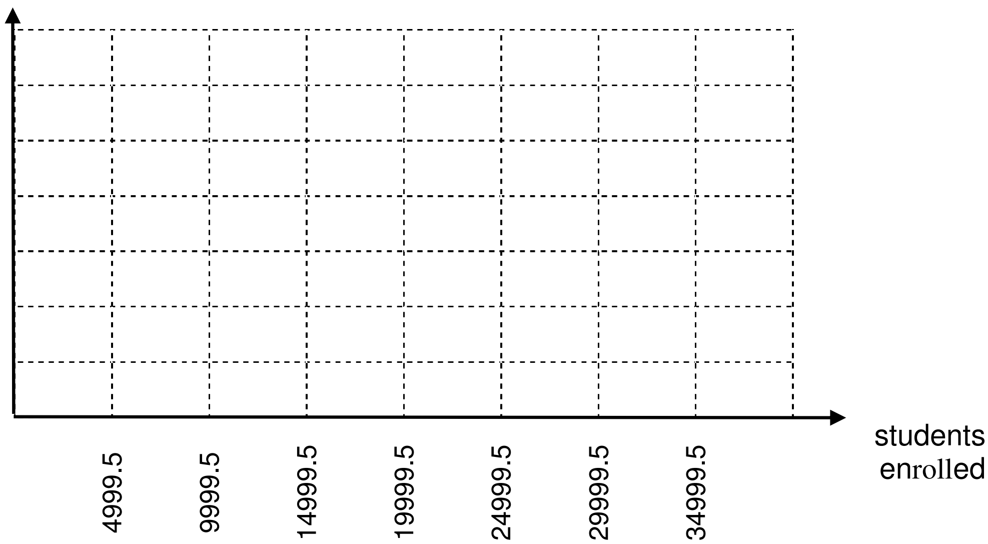
Interval (Class Limits)	Class Boundaries	Frequency	Relative Frequency	Cumulative Relative Frequency
5000-9999				
10000-14999				
15000-19999				
20000-24999				
25000-29999				
30000-34999				

NUMERICAL SUMMARIES & GRAPHICAL DISPLAYS OF QUANTITATIVE DATA: HISTOGRAMS AND DISTRIBUTIONS

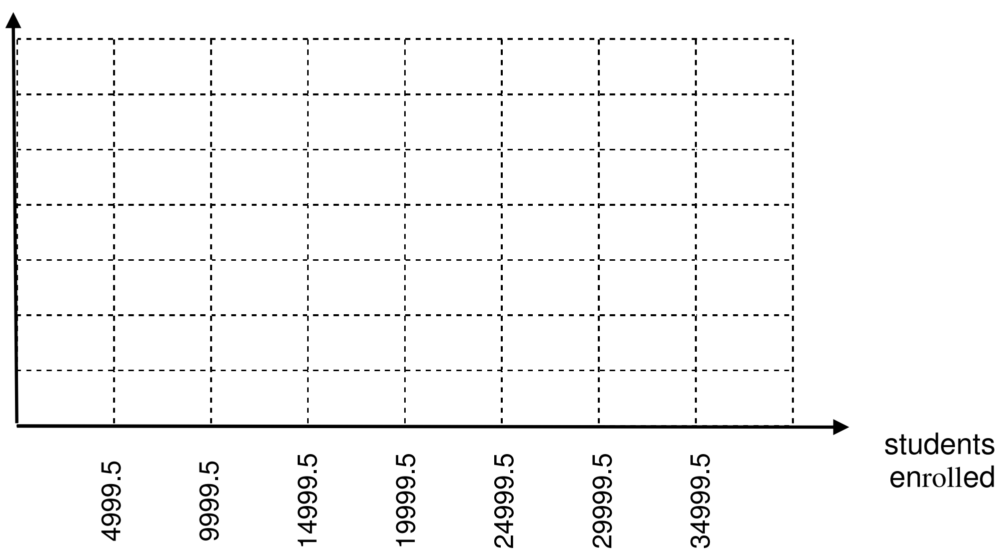
A HISTOGRAM is a bar graph displaying quantitative (numerical) data

Consecutive bars should be touching. There should not be a gap between consecutive bars.
A "gap" should occur only if an interval does not have any data lying in it.
Vertical axis can be frequency or can be relative frequency.

4b. Draw a **frequency histogram**, using the graph grid below.
Draw it accurately and label and scale both axes.



4c. Draw a **relative frequency histogram**, using the graph grid below.
Draw it accurately and label and scale both axes.



GRAPHICAL DISPLAYS OF QUANTITATIVE DATA: STEM AND LEAF PLOTS

Each data value is split into a stem and leaf using place value.
A key indicating the place value representation by the stem and leaf should be shown.

EXAMPLE 5:

Suppose that a random sample of 18 mathematics classes at a community college showed the following data for the number of students enrolled per class:

Raw Data:	**37, 40, 38, 45, 28, 60, 42, 42, 32, 43, 36, 40, 82, 42, 39, 36, 60, 25**
Sorted Data:	**25, 28, 32, 36, 36, 37, 38, 39, 40, 40, 42, 42, 42, 43, 45, 60, 60, 82**

EXAMPLE 6:
Practice - Try It!

The table shows the number of baseball games won by each American League Major League Baseball Team in the 2010 regular season.

2010 Regular Season	Games Won
Tampa Bay Rays	96
New York Yankees	95
Boston Redsox	89
Toronto Blue Jays	85
Baltimore Orioles	66
Minnesota Twins	94
Chicago White Sox	88
Detroit Tigers	81
Cleveland Indians	69
Kansas City Royals	67
Texas Rangers	90
Oakland A's	81
LA Anaheim Angels	80
Seattle Mariners	61

Games Won (Sorted Data)
61
66
67
69
80
81
81
85
88
89
90
94
95
96

Construct a stem and leaf plot:

EXAMPLE 7: Read the data from this stem and leaf:
Weights of **18** randomly selected packages of meat in a supermarket, in pounds.

1	389999
2	00011268
3	27
4	
5	0
6	2

Leaf Unit = .1
Stem Unit = 1
1|9 = 1.9

What is the weight of the smallest package?______

What is the weight of the largest package? ______

How many packages weigh at least 2 but less than 4 pounds? _____

How many packages weigh at least 4 but less than 5 pounds? _____

How many packages weigh at least 5 pounds? ______

EXAMPLE 8: Read the data from this stem and leaf:
Number of students at each of **18** elementary schools in a city

1	389999
2	00011268
3	27
4	
5	0
6	2

Leaf Unit = 10
Stem Unit = 100
1|9 = 190

How many students in the smallest school? ______

How many students in the largest school? ______

Read back several data values from the stem and leaf plot.
Do you notice anything interesting about the data?
Do you think that these numbers could represent the actual raw data or might they have been altered in some way?

DESCRIPTIVE STATISTICS:
MEASURES OF RELATIVE STANDING: PERCENTILES & QUARTILES

The **P^{th} percentile** is divides the data between the lower P% and the upper (100 – P)% of the data:

P% of data values are less than (or equal to) the P^{th} percentile

(100-P)% of data values are greater than (or equal to) the P^{th} percentile

EXAMPLE 9: ***Interpreting Quartiles and Percentiles***

A class of 20 students had a quiz in the sixth week of class. Their quiz grades were:

2 5 10 10 12 12 13 14 14 14 15 15 17 17 18 18 20 20 20 20

a. The 40^{th} percentile is a quiz grade of 14.

40% of students had quiz grades of 14 or less. 60% of students had quiz grades of 14 or more

2 5 10 10 12 12 13 14 14 14 15 15 17 17 18 18 20 20 20 20

$P_{40} = 14$

b. The 20^{th} percentile is a quiz grade of 11. Write a sentence that interprets (explains) what this means in the context of the quiz grade data.

Chapter 2 Practice #3 in textbook (online at http://cnx.org/content/m18845/latest/) provides practice understanding percentiles and guidelines to writing interpretations. Try it!!

"Special" Percentiles: First Quartile Q1 Median Third Quartile Q3

Your calculator can find these special percentiles using 1-variable statistics(Q1, Med, Q3).

The INTERQUARILE RANGE (IQR) is the difference between the third and first quartiles. The IQR measures the spread of the middle 50% of the data : IQR = Q3 – Q1

c. Find the Interquartile Range Q1 = ______ Q3 = ______ IQR = ___________________

<u>Finding summary statistics on your TI-83,84 calculator</u>

Enter data into the statistics list editor: STAT "EDIT" press enter

If <u>not</u> using a frequency list*:* Put data into list L1, press ENTER after each data value

2nd QUIT to exit stat list editor <u>after</u> you have entered data, checked it and corrected errors.

One Variable Summary Statistics: STAT "CALC" 1 *for 1 – Var Stats* 2nd L1 ENTER.

If data is in a different list than L1, indicate the appropriate listname instead of L1

If using a frequency list*:* Put data into list L1, frequencies into list L2, press ENTER after each data value

2nd QUIT to exit stat list editor <u>after</u> you have entered data, checked it and corrected errors.

One Variable Summary Statistics: STAT "CALC" 1 *for 1 – Var Stats* 2nd L1 , 2nd L2 ENTER

order of lists should be data value list, frequency list

Estimating Percentiles From Cumulative Relative Frequency

(using the method from Collaborative Statistics, B. Illowsky & S. Dean, www.cnx.org)

EXAMPLE 10: 2 5 10 10 12 12 13 14 14 14 15 15 17 17 18 18 20 20 20 20

x	Frequency	Relative Frequency	Cumulative Relative Frequency
2	1	1/20 =0.05	0.05
5	1	0.05	0.10
10	2	2/20 = 0.10	0.20
12	2	0.10	0.30
13	1	0.05	0.35
14	3	3/20 = 0.15	0.50
15	2	0.10	0.60
17	2	0.10	0.70
18	2	0.10	0.80
20	4	4/20 = .20	1.00

Sort data into ascending order and complete the cumulative relative frequency table.
Do NOT group the data into intervals. Each data value is on its own line in the table.

To estimate the p^{th} percentile, move down the cumulative relative frequency column looking for the decimal value of p for the percentile.

- **PROCEDURE: IF YOU PASS BEYOND THE DECIMAL VALUE OF p:**
  the p^{th} percentile is the data value (x) column at the first line in the table BEYOND the value of p
  EXAMPLE: Find the 40^{th} percentile:
  Go down right hand column looking for 0.40. You don't find 0.40 exactly.
  You pass 0.40 between 0.35 and 0.50.
  The 40^{th} percentile is the x value for the line at which you first pass 0.40.
  The 40^{th} percentile is 14
- **PROCEDURE: IF YOU FIND THE EXACT DECIMAL VALUE OF p:**
  the p^{th} percentile is the average of the data (x) value in that line and in the next line of the table
  EXAMPLE: Find the 20^{th} percentile:
  Go down right hand column looking for 0.20. You find 0.20 exactly, on the line where x = 10.
  The 20^{th} percentile is the average of the x values on that line (10) and on the line below it (12)
  The 20^{th} percentile is (10+12)/2=11

EXAMPLE 11 : *TRY IT!* Use the table to find the first and third quartiles and the median.

WHY DO WE DO IT THIS WAY?

This method finds the median correctly, for even or odd numbers of data values.
Then we use the same method for all other percentiles.

For the median: The median is 14.5. When there are an even number of data values, the median is the average of the two middle values: 14 and 15. Using the table to find the 50^{th} percentile, we see 0.50 exactly in the table; the procedure tells us to average the x value, 14, and the next x value, 15. This gives 14.5 as the 50^{th} percentile.
If you did not average, but used the x value for the line showing 0.50, you would use 14 as the median which is not correct.

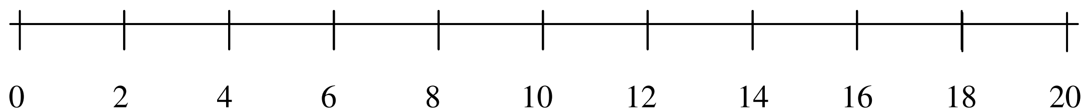
GRAPHICAL REPRESENTATION OF DATA: BOXPLOTS

EXAMPLE 12 : ***Creating Box Plots using the 5 number summary from 1–Var Stats on your calculator***

A class of 20 students had the following grades on a quiz during the 6th week of class

2 5 10 10 **12** **12** 13 14 14 **14** **15** 15 17 17 **18** **18** 20 20 20 20

Find the 5 number summary and draw a boxplot for the quiz grade data.
The box identifies the IQR. The lines (whiskers) extend to the minimum and maximum values.
Mark the median inside the box.



Boxplots are easy to do by hand once you have found the 5 number summary. If you want to learn how to create a boxplot on your calculator, refer to the technology section in the appendix of the textbook or to the online calculator handout instructions for your model of calculator.

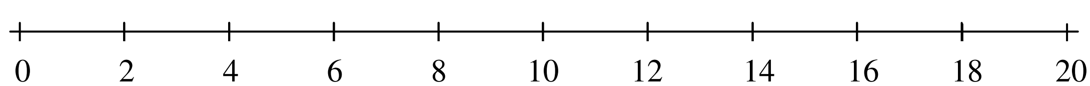
EXAMPLE 13: Find the 5 number summary and draw the boxplot

X	Frequency
3	40
5	25
6	11
7	3
10	2

EXAMPLE 14 ***(optional):***

Sometimes you may see a boxplot in which the whiskers (lines) are extended only until the lower and upper fences and any data that is more extreme than the fences are indicated by a dot • (or ∗, + or ∘).

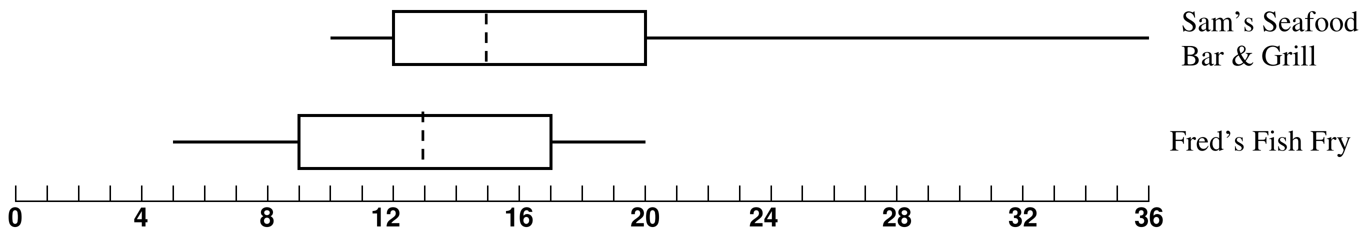
It is more complicated to construct, but has the advantage that outliers are easily identified visually.



GRAPHICAL REPRESENTATION OF DATA: BOXPLOTS

EXAMPLE 15: ***Interpreting Box Plots***

The boxplots represent data for the amount a customer paid for his food and drink for random samples of customers in the last month at each of two restaurants



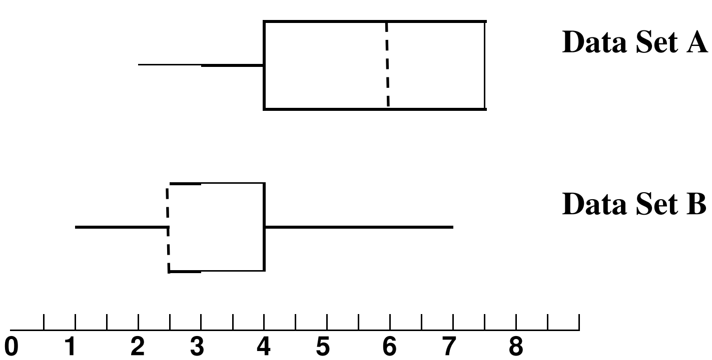
Find these values by reading the boxplot.

Sam's: Min _______ Q1 _______ Median _______ Q3 _______ Max _______ IQR________

Fred's: Min _______ Q1 _______ Median _______ Q3 _______ Max _______ IQR________

Use the boxplots to compare the distributions of the data for the two restaurants. Look at the statistics for the center, quartiles, and extreme values, and the spread of the data. Discuss differences and/or similarities you see regarding the <u>location</u> of the data, the <u>spread</u> of the data, the <u>shape</u> of the data, and the existence of <u>outliers</u>.

EXAMPLE 16: What is strange about these boxplots?
Explain what is "strange" and what it means in each boxplot



DESCRIPTIVE STATISTICS: Identifying Outliers Using Quartiles & IQR

Outliers are data values that are unusually far away from the rest of the data.

We calculate values called "fences" to decide if a data value is close to or far from the rest of the data. Any data that is not between the fences (inclusive) is considered an outlier.

Lower Fence: Q1 – 1.5*IQR **Upper Fence: Q3 + 1.5*IQR**

Outliers should be examined to determine if there is a problem (perhaps an error) in the data.

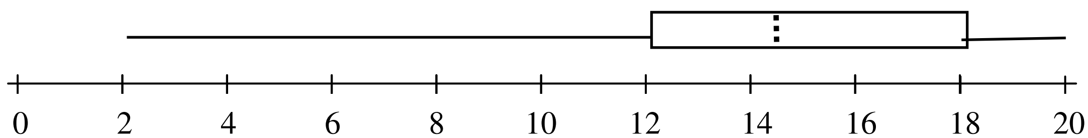
If the outlier is due to an error that can not be corrected, or has properties that show it should not be part of the data set, it can be removed from the data.

If the outlier is due to an error that can be corrected, the corrected data value should remain in the data.

If the outlier is a valid data value, the outlier should be kept in the data set.

Each situation involves individual judgment depending on the situation.

OUTLIER AND BOXPLOTS: Graphical View:



The IQR is the length of the box.

- The line from the box to the lowest data value is longer than 1½ times the length of the box. This indicates that there are outliers at the low end of the data.
- The line from the box to the highest data value is shorter than 1½ times the length of the box. This shows that there are not any outliers at the high end of the data.

OUTLIERS: Calculating the Fences and Identifying Outliers

For a quiz, exam, or graded work, you must know be able to show your work doing the calculations to find the fences and explain your conclusion.

EXAMPLE 17: For the quiz grade data, find the lower and upper fences and identify any outliers.

2 5 10 10 12 12 13 14 14 14 15 15 17 17 18 18 20 20 20 20

IQR = **Lower Fence: Q1 – 1.5(IQR) =**

Upper Fence: Q3 + 1.5(IQR) =

Are there any outliers in the data? Justify your answer using the appropriate numerical test.

In Math 10, we will find outliers by finding the fences using Q1, Q3 and the IQR as above This method is usually considered appropriate for data sets of all shapes.

NOTE: ***There are many statistical methods of indentifying outliers or unusual values.*** *The different methods sometimes produce different results.*

For mound-shaped and symmetric data*, statisticians may flag outliers by finding values that are further than 2 (or further than 3) standard deviations away from the mean. This method is not generally appropriate for data distributions with other shapes. This method is based on the "Empirical Rule" and the "Normal Probability Distribution" that we will study later in this course.*

Chemistry students often learn another method called a "Q-test".

A statistics professor at UCLA wrote a 400+ page book about different methods of finding outliers!

DESCRIPTIVE STATISTICS: MEASURES OF CENTRAL TENDENCY (CENTER)

Mean = Average = $\dfrac{\text{sum of all data values}}{\text{number of data values}}$ Symbols: Sample Mean: $\overline{X}$ Population Mean μ

Median = Middle Value (if odd number of values) OR Average of 2 middle values (if even number of values)

Mode = most frequent value

EXAMPLE 18: The table shows the lowest listed ticket prices in the San Jose Mercury News for 15 major Bay Area concerts during one randomly selected week in the summer. Consider this to be a sample of all concerts.

35	35	45	54	45	33	35	40	38	48	75	89	35	45	44

Ticket Price Data Sorted into Order

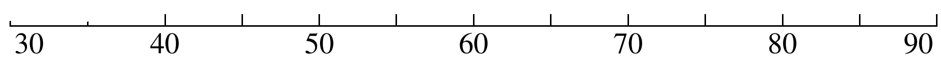
33	35	35	35	35	38	40	44	45	45	45	48	54	75	89

Find the mean

Find the median

Find the mode

Draw a dotplot of the data.



Which value should be used as the most appropriate measure of the center of this data?

EXAMPLE 19:

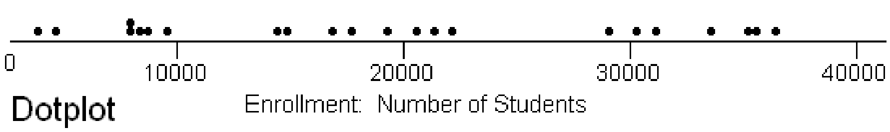
CSU Enrollment for Fall 2009 : These data are for all 22 CSU "non-specialized" campuses.

Find the **<u>mean</u>** (average) number of students

Find the **<u>median</u>** number of students

Which value should be used as the most appropriate measure of the center of this data?

CSU Campus	2009 Enrollment
Channel Islands	3,862
Monterey Bay	4,688
Humboldt	7,954
Bakersfield	8,003
Sonoma	8,546
Stanislaus	8,586
San Marcos	9,767
Dominguez Hills	14,477
East Bay	14,749
Chico	16,934
San Bernardino	17,852
San Luis Obispo	19,325
Los Angeles	20,619
Fresno	21,500
Pomona	22,273
Sacramento	29,241
San Francisco	30,469
San Jose	31,280
San Diego	33,790
Northridge	35,198
Long Beach	35,557
Fullerton	36,262



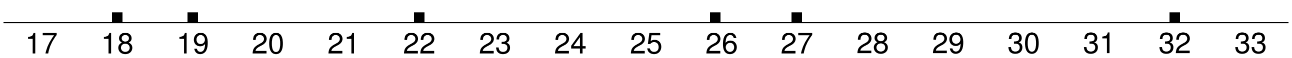
DESCRIPTIVE STATISTICS: MEASURES OF VARIATION (SPREAD)

EXAMPLE 20: Ages of students from two classes Random sample of 6 students from each class

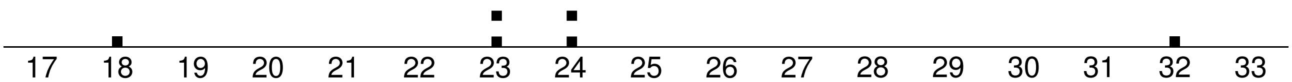
	Age Data						Mean	Range	Standard Deviation
Sample from Class 1	18	19	22	26	27	32	24	14	5.33
Sample from Class 2	18	23	23	24	24	32	24	14	4.52

Range = Maximum Value – Minimum Value = ______ –_____ =_____

DOTPLOT: Sample from Class 1



DOTPLOT: Sample from Class 2



Based on the dotplots, does one sample appear to have more variation than the other sample?___________

The **Standard Deviation** measures variation (spread) in the data by finding the distances (deviations) between each data value and the mean (average).

Sample from Class 1:			
x	$\bar{x}$	$x-\bar{x}$	$(x-\bar{x})^2$
18	24		
19	24		
22	24		
26	24		
27	24		
32	24		
		$\sum_{\text{all data}}(x-\bar{x})^2 =$	
Sample Variance: $S^2 = \frac{\sum(x-\bar{x})^2}{n-1} =$			
Sample Standard Deviation: $S = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} =$			

Sample from Class 2: ***Practice - Try It!***			
x	$\bar{x}$	$x-\bar{x}$	$(x-\bar{x})^2$
		$\sum_{\text{all data}}(x-\bar{x})^2 =$	
Sample Variance: $S^2 = \frac{\sum(x-\bar{x})^2}{n-1} =$			
Sample Standard Deviation: $S = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} =$			

Calculate the standard deviation for the sample from Class 2 for homework, if not done in class.

DESCRIPTIVE STATISTICS: MEASURES OF VARIATION (SPREAD)

Use Standard Deviation as the most appropriate measure of variation	**SAMPLE STANDARD DEVIATION**	**POPULATION STANDARD DEVIATION**
	n individuals in sample sample mean is $\bar{x}$ $S=\sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$ If using sample data, use Sx from your calculator's 1VarStats	*N* individuals in population population mean is μ $\sigma=\sqrt{\frac{\sum(x-\mu)^2}{N}}$ If using population data, use **σ**x from your calculator's 1VarStats

EXAMPLE 21: A class of 20 students has a quiz every week. All students in the class took the quizzes.

For the sixth week quiz, the grades are

2 5 10 10 12 12 13 14 14 14
15 15 17 17 18 18 20 20 20 20

x	Frequency
2	1
5	1
10	2
12	2
13	1
14	3
15	2
17	2
18	2
20	4

For the seventh week quiz, the grades are

1 8 8 12 13 13 13 14 14 14
14 14 15 15 17 17 18 18 18 20

x	Frequency
1	1
8	2
12	1
13	3
14	5
15	2
17	2
18	3
20	1

a. Use your calculator to find the mean, median and standard deviation for each quiz.
Which symbol is appropriate to use for the mean in this example: $\bar{x}$ or μ ? Why?
Which standard deviation is appropriate to use in this example: s or σ? Why?

6th week quiz: Mean ___ = ______ Median = ______ Standard Deviation ____ = _______

7th week quiz: Mean ___ = ______ Median = ______ Standard Deviation ____ = _______

b. Which week's quiz exhibits more variation in the quiz grades? Justify your answer numerically.

c. Which week's quiz exhibits more consistency in the quiz grades? Justify your answer numerically

d Find the variance for each week's quiz grades:

6th week quiz: ________________________ **7th week quiz:** __________________________

DESCRIPTIVE STATISTICS: Measures of Relative Standing: Z-SCORES

"z-score" tells us how far away a data value is from the mean, measured in "units" of standard deviations
It describes the location of a data value as "how many standard deviations above or below the mean"

$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}} = \frac{x-\mu}{\sigma} \quad or \quad \frac{x-\bar{x}}{s}$$

In our textbook this is also noted as #of STDEVs

EXAMPLE 22: In the 6th week of class, the 20 students had the quiz grades below. Anya's quiz grade was 18.

2 5 10 10 12 12 **13** 14 14 14 15 15 17 17 18 **18** 20 20 20 20 **μ =14.3 σ = 4.8**

$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}} = \frac{x-\mu}{\sigma} = \frac{18-14.3}{4.8} = \frac{3.7}{4.8} = 0.77$$

Anya's quiz grade was 3.7 *points* above average but it was 0.77 *standard deviations* above average.

Interpretation of Anya's z-score for the quiz:
Anya's quiz grade of 18 points is 0.77 standard deviations above the average quiz grade of 14.3

EXAMPLE 19: In the 8th week of class, the 20 students had the exam grades below: Anya's exam grade was 90

44 52 56 59 **62** 65 70 71 72 74 74 75 77 79 84 85 **90** 91 94 100 μ = 73.7 σ = 14.25

Find and interpret Anya's z-score for the exam:

Did Anya perform better on the quiz or the exam when compared to the other students in her class?
Use the z-scores to explain and justify your answer.

EXAMPLE 23: In the same class as Anya, Bob's quiz grade was 13 points and his exam grade was 62 points.

Find and interpret Bob's z-score for the quiz.

Did Bob perform better on the quiz or the exam when compared to the other students in his class?
Use the z-scores to explain and justify your answer.

GUIDELINE: Writing a sentence interpreting a z-score in the context of the given data:

The (*description of variable*) of (*data value*) is |*z-score*| standard deviations (*above or below*) the average of (*value of the mean*)

Write absolute value of z (*drop the sign*)

Use
above if z score > 0
below if z score < 0

Z-Scores Continued

EXAMPLE 24: Z-scores for quiz grades on week 6 quiz for 4 students in the class:

Student	Anya	Bob	Carlos	Dan
Z-score			1.19	– 0.90

Based on the Z-scores, arrange the students quiz grades in order. Which is best? Which is worst?

_______________ _______________ _______________ _______________

EXAMPLE 25: Working Backwards from Z-score to Data Value

$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}} = \frac{x-\mu}{\sigma} \quad or \quad \frac{x-\bar{x}}{s}$$ can be solved for "x=":

A data value can be expressed as $x = \text{mean} + (\text{z-score})(\text{standard deviation}) = \bar{x} + z\,s \quad or \quad \mu + z\sigma$

For the week 6 quiz, $\mu = 14.3$ and $\sigma = 4.8$. Find the quiz scores for Carlos and Dan:

Carlos: $z = 1.19$ x = ____

Dan: $z = -0.90$ x =____

Are high or low z-scores good or bad? It depends on the context of the problem.
Read each problem carefully. Think about the context and the meaning of the numbers for that problem.

Positive z-scores correspond to numbers that are larger than the average.
Higher than average is good for exam scores and salaries
Higher than average is bad for airline ticket costs or waiting time for a bus to arrive.
High z scores are good for race speeds (fast) but bad for race times (slow),
Negative z-scores correspond to numbers that are smaller than the average.
Lower than average is bad for exam scores and salaries.
Lower than average is good for airline ticket costs or waiting time for a bus to arrive.
Small z scores are bad for race speeds (slow) but good for race times (fast),
In some contexts, no value judgment applies; such as the number of children in a family

EXAMPLE 26: The air at an industrial site is tested for a sample of 30 days to measure the level of two pollutants: A and B. (A and B are measured in different units, have different "safe" levels, and different effects on public health, so are not directly comparable.)

Suppose that for today's pollution readings:

The level of pollutant A is 0.5 standard deviations below its average level: z = ______

The level of pollutant B is 0.8 standard deviations below its average level: z = ______

a. Compare today's pollution levels for A and B to the average readings for the 30 day sample at this site. Which of today's pollutant levels would be considered better for this site? Explain.

Today the level for pollutant _____ is better because

b *Practice: Working Backwards:* Suppose that the sample averages and standard deviations are
Pollutant A: $\bar{x}$ = 47 parts per billion, s = 4 Pollutant B: $\bar{x}$ = 10 micrograms per m^3, s = 1.5 ;
Find the actual levels for pollutants A and B.

(Note: Data underlying this example: h*ttp://www.epa.gov/air/criteria.html* The National Ambient Air Quality Standards , specify average "safe levels" that must be maintained in order to protect public health for various pollutants:
A: Nitrogen Dioxide NO_2 : 53 parts per billion ; B: Particulate Matter $PM_{2.5}$: 15 micrograms per m^3.)

Statisticians may use the values that are two or three standard deviations away from the mean as guidelines for data values that are "extreme".

EXAMPLE 27:
In the 6th week of class, the 20 students had the quiz grades below.

2 5 10 10 12 12 13 14 14 14 15 15 17 17 18 18 20 20 20 20 μ =14.3 σ = 4.8

Find the values that are 2 standard deviations above and below the mean.

Are there any data values that are NOT between these values?

EXAMPLE 28:
A food processing plant fills cereal into boxes that are labeled to contain 20 ounces of cereal.

A machine fills boxes with an average of 20.6 ounces of cereal and a standard deviation is 0.2 ounces.

For quality assurance, the food processing plant manager needs to monitor how much cereal the boxes actually contain; each day randomly selected of boxes of cereal are weighed.

a. Find the values that are three standard deviations above and below the average.

b. Why might the manager be concerned if there are boxes of cereal with weight less than the value that is 3 standard deviations below average?

c. Why might the manager be concerned if there are boxes of cereal weighing more than 3 standard deviations above average?