

# HYPOTHESIS TEST NOTES

Page 1: Formulating Hypotheses

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Page 3: Type I and Type II Errors

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Pages 6,7,8: Lecture examples of complete hypothesis tests

*Hypothesis Test Powerpoint – available online reviews major concepts and explains details of the mathematical steps and writing interpretations*

## Hypothesis Test

- procedure that allows us to ask a question about an unknown population parameter
- uses sample data to draw a conclusion about the unknown population parameter.

## Steps to perform a Hypothesis Test

Step 1: Planning the test: Formulate questions as hypotheses

AND set criteria for how to draw a conclusion from the data

Step 2: *In real life: Gather Data:* Select random sample(s) and collect data.

*In HW or exam problem:* Examine the given data and information, and determine how to perform test (which distribution and type of test to use)

Step 3: Analyze sample data:

Step 4: Decide what the data analysis shows

Step 5: Interpret the decision in the context of the problem.

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## Step 1: Set up hypotheses that ask a question about the population

by writing two opposite statements about the possible value of the parameter.

**Ho: Null hypothesis:** The assumption about the population parameter that will be believed unless it can be shown to be wrong beyond a reasonable doubt

**Ha: Alternate hypothesis:** The claim about the population parameter that must be shown correct "beyond a reasonable doubt" to believe that it is true.

**Example A:** A new drug is being tested for safety.

Ho: Null hypothesis: The new drug is not safe.

Ha: Alternate hypothesis: The new drug is safe.

The hypothesis test should be set up so that the drug must be proven safe in order for it to be used.

**Example B:** To be considered "fat-free", regulations require that a serving of salad dressing must contain less than  $\frac{1}{2}$  gram of fat. A salad dressing manufacturer must be able to show that its salad dressing satisfies these guidelines in order to put the words "fat-free" on its label. (*one "serving" is 2 tablespoons of salad dressing*)

Ho: Null hypothesis:  $\mu \geq 0.5$  The salad dressing has an average  $\frac{1}{2}$  gram of fat, or more, per serving.

Ha: Alternate hypothesis:  $\mu < 0.5$  The salad dressing has less than  $\frac{1}{2}$  gram of fat, on average, per serving.

The hypothesis test should be set up so that the manufacturer must prove that the salad dressing meets satisfies the regulations in order to be able to call it "fat-free"

In real world studies, statisticians design the hypothesis test so that the outcome that needs to be proved is the alternate hypothesis.

### **Math 10 RULE FOR HYPOTHESES**

**Null hypothesis Ho must contain equality of some type: = ≤ or ≥**

**Alternate hypothesis Ha must contain a pure inequality. ≠ > or <**



## Hypothesis Tests: Correct Decisions and Errors in Decisions

In a hypothesis test, we decide about the hypotheses, based on the strength of the evidence in the sample data. The sample data may lead us to make a correct decision or sometimes make a wrong decision. We want to make decisions in a way that minimizes the chance of making a wrong decision.

IN REALITY:	Null Hypothesis is TRUE	Null Hypothesis is FALSE Alternate Hypothesis is true
DECISION: Based on sample data we DECIDE NOT TO REJECT null hypothesis	<b>Correct Decision</b>	<b>Type II Error: Wrong Decision</b> Probability is $\beta$
Based on sample data we DECIDE TO REJECT null hypothesis	<b>Type I Error: Wrong Decision</b> Acceptable probability (risk) is $\alpha$ $\alpha$ is called <b>significance level</b>	<b>Correct Decision</b> Probability is $1 - \beta$ $1 - \beta$ is called the <b>power</b>

### Type I Error:

**Concluding (based on sample data) in favor of the alternate hypothesis when in reality the null hypothesis is true**  
*Rejecting the null hypothesis when in reality the null hypothesis is true*

The probability you are willing to risk of making a Type I error is denoted by  $\alpha$  (Greek letter alpha).  $\alpha$  is called the (pre-determined) **significance level** of the hypothesis test.

The significance level  $\alpha$  should be small: a low risk of incorrectly rejecting the null hypothesis if it is really true

### Type II Error:

**Concluding (based on sample data) in favor of the null hypothesis when in reality the alternate hypothesis is true**  
*Failing to reject the null hypothesis when in reality the null hypothesis is false*

The probability of Type II error is denoted by  $\beta$  (Greek letter beta).

$1 - \beta$  is called the **power** of the hypothesis test.

The power of a hypothesis test should be large: large probability of correctly rejecting a false null hypothesis

### Example C Revisited: (see page 2 for the statement of this problem)

$p$  = true population proportion of all people with this knee injury who would be cured if they had this knee surgery

**Ho:  $p \leq .60$**

**Ha:  $p > .60$**

A Type I error would be to **conclude** that this surgery cures more than 60% of **all** knee injuries of this type, **when in reality** it cures 60% or fewer of **all** such injuries.

A Type II error would be to **conclude** that this surgery cures at most 60% of **all** knee injuries of this type, **when in reality** it cures more than 60% of **all** such injuries.

*Some of the examples on page 2 will be used in class to interpret errors; Catalyst website has a link to all answers.*

### GUIDELINES to Interpreting Type I and Type II Errors in context:

The interpretation of a Type I or Type II Error must have 2 parts: CONCLUSION (*decision*) and REALITY  
In describing the "error", the conclusion and reality do NOT agree with each other  
(because if they agree, then there would be no error –it would be a correct decision)

Interpretation must be worded in context of the problem AND:

- must have descriptions in words of the conclusion (decision) AND of the situation in reality
- must correctly refer to average or proportion or percent
- must refer to the population (not the sample)
- wording must accurately reflect the equality and inequality symbols in the hypotheses.

### **Interpretation of Type I Error:**

A Type I error is to CONCLUDE (*state the ALTERNATE hypothesis in the words of the problem*)  
WHEN IN REALITY (*state the NULL hypothesis in the words of the problem*)

### **Interpretation of Type II Error:**

A Type II error is to CONCLUDE (*state the NULL hypothesis in the words of the problem*)  
WHEN IN REALITY (*state the ALTERNATE hypothesis in the words of the problem*)

## Rare Events and Making Decisions

We have an assumption about a property of a population.

We select a sample.

- Suppose our sample data has properties that are extremely unlikely to occur if the population actually has the properties we assumed. We would then conclude that the assumption is not correct.
- Suppose our sample data has properties that are reasonably likely to occur if the population actually has the properties we assumed. This would not give us any reason to doubt the assumption.

### Example D

You and a friend go to a party. There is a big clean black barrel containing cans of soda in ice water.

You can't see into it. Somebody who you don't know at the party tells you the barrel contains

99 cans of Pepsi and one can of Sprite. You don't know who he is or where he got that information.

You want a can of Sprite and you are the first one to pick a soda from the barrel.

IF what you were told is true, then there is only a small 1/100 chance that you will pick the can of Sprite.

You figure it will not happen. BUT it happens.

You reach in and get a can of Sprite. It was possible but very unlikely to happen.

You tell your friend that maybe you were just lucky but that you really think the guy who told you about the 99 cans of Pepsi and 1 can of Sprite was wrong about that.

Although your data (can of Sprite) does not "prove" that his information about the cans of soda was wrong, it gives very strong evidence that his information is likely to be wrong.

### Example C Revisited: Suppose that a hospital is testing a new surgery for a certain type of knee injury.

The hospital's surgery review board needs to decide whether to approve this type of surgery.

The surgery review board has decided that they will approve this surgery if a clinical trial shows that the true population cure rate for this surgery would be more than 60%. Otherwise they will not approve it.

Population parameter:  $p$  = true population cure rate for this surgery

Random Variable:  $P'$  = cure rate for a sample of patients having this surgery

$H_0$ :  $p \leq .60$

$H_a$ :  $p > .60$

- Suppose the new surgery is tested on 200 patients.
- Suppose the sample proportion of people who are cured is  $p'=0.90$ , a 90% cure rate.  
A 90% cure rate is much higher than the 60% cure rate in the hypotheses.  
We conclude that the null hypothesis  $H_0: p \leq .60$  is not likely to be true  
We conclude that the alternate hypothesis  $H_a: p > .60$  is true
- Suppose the sample proportion of people who are cured is  $p'=0.61$ , a 61% cure rate.  
The sample cure rate is very close to the 60% cure rate criteria in the hypotheses.  
We don't think the sample cure rate is exactly the population cure rate. Sample statistics vary somewhat from the true population value due to randomness (sampling error).  
So we're not sure that for the population the cure rate would actually be larger than 60%.  
We think its also possible that the population cure rate may actually be 60% (or slightly lower), because our sample value of 61% is so close to 60%.  
We do not have strong enough evidence in the sample data to conclude that  $H_a: p > .60$  is true

But what if  $p' = 0.75$  or  $0.70$  or  $0.65$ ? How far away from the null hypothesis should the sample value be in order to reject the null hypothesis and assume the alternate hypothesis is true?

We will use probability calculations to determine where to draw the "dividing line" to decide if a sample is close to or far from the null hypothesis.

**Example C continued:** Population parameter:  $p$  = true population cure rate for this surgery  
 Random Variable:  $P'$  = cure rate for a sample of patients having this surgery  
 **$H_0: p \leq .60$     $H_a: p > .60$**

Suppose that in a sample of 200 patients, 130 of them are cured by this surgery:  $p' = 130/200 = 0.65$

If the null hypothesis  $H_0$  is true, we expect to get a sample statistic "close to" the value of the population parameter  $H_0$ . (Because  $H_0$  contains "=" in some form.)

We need to determine whether our sample proportion  $p' = 0.65$  is "close to" (consistent with) or "far from" (not consistent with) the hypothesized proportion, 0.60, in  $H_0$

To do this we examine probability of getting a sample that "looks like ours" if the null hypothesis is true.

If that probability is small, it indicates that our sample is not consistent with the null hypothesis, but is strong evidence in support of the alternate hypothesis, leading us to reject the null hypothesis.

We need to set a criteria for "what is a small probability?"

A Type I Error would be to decide to approve the surgery, believing it has a cure rate of more than 60%, ( $p > .60$ ) if in reality surgery is not effective, having a cure rate that is not more than 60% ( $p \leq .60$ )

What risk of making a Type I error are we willing to accept?

The risk we are willing to accept of making a Type I error is the significance level  $\alpha$ .

The significance level  $\alpha$  is our criteria for "what is a small probability?"

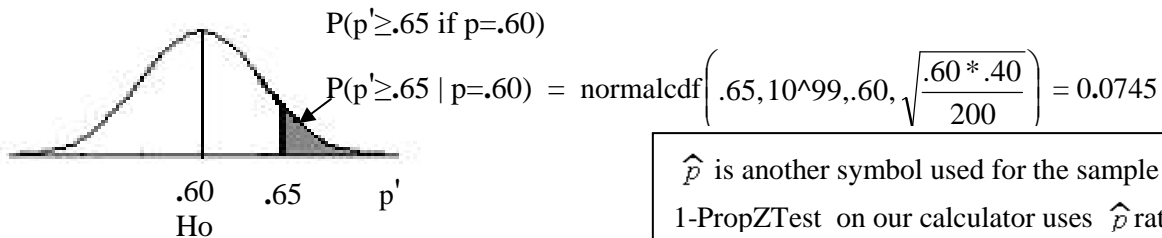
Suppose we are willing to accept only a 2% risk of making a Type I error ( $\alpha = 0.02$ )

We don't want to expose people to medical risks of surgery unless we are confident that it is effective.

We want a small probability of erroneously approving surgery if it is in reality not effective.

**Calculate the probability of getting a sample at least as far from the null hypothesis as our sample is. This is called the p value.** Since our alternate hypothesis says  $>$ , we use the right tail of the distribution

If  $H_0$  is true, the random variable  $P'$  is distributed  $N\left(0.60, \sqrt{\frac{0.60 * 0.40}{200}}\right)$



$\hat{p}$  is another symbol used for the sample proportion  
 1-PropZTest on our calculator uses  $\hat{p}$  rather than  $p'$

**Compare the p value to the significance level  $\alpha$**

Is the probability of making a Type I error for this sample smaller than our risk level?

If no, we do not contradict the null hypothesis. If yes, we decide that we believe alternate hypothesis.

**Is the p value smaller than the significance level? No.**

Our p value of 0.0745 is greater than the significance level,  $\alpha$ , of 0.02

$$p \text{ value} > \alpha \qquad 0.0745 > 0.02$$

We decide that the sample does not provide strong enough evidence to contradict the null hypothesis.

**Decision:** DO NOT REJECT THE NULL HYPOTHESIS (because  $p \text{ value} > \alpha$ )

**Conclusion:** At a 2% level of significance the sample data do NOT provide strong enough evidence to conclude that the true cure rate for this surgery is greater than 60%. (NOTE: This means the new surgery will NOT be approved. The review board is not convinced that it is effective enough.)

**DECISION RULE: If  $p \text{ value} < \alpha$ , REJECT  $H_0$**   
**If  $p \text{ value} \geq \alpha$ , DO NOT REJECT  $H_0$**

**CONCLUSION:** At a (*state  $\alpha$  as %*) level of significance, the sample data *DO / DO NOT* provide strong enough evidence to conclude that (*state in words what the alternate hypothesis says in context of the problem*)

**TRY IT:** Suppose in a sample of 200 patients, 138 of them are cured by this surgery:  $p' = 138/200 = 0.69$

**EXAMPLE E: Hypothesis Test of Population Mean  $\mu$  when Population Standard Deviation  $\sigma$  is KNOWN**

A computer company has a new "power saver feature" that it claims will extend the average working time for its laptop batteries to longer than 240 minutes (4 hours) before they need to be charged. The company management is willing to invest in producing and marketing the power-saver laptop if its engineers can show strong evidence in the sample data to conclude that the true average working time for all batteries used in power-saver laptops is longer than 240 minutes.

*(Testing is done under conditions conforming to industry standards for brightness and typical laptop workload.)*

Data for sample of 30 power-saver laptops: time, in minutes, that batteries work until needing recharging  
244 223 243 299 212 190 271 254 295 264 258 255 258 281 234  
237 230 281 250 257 263 282 264 259 256 265 269 191 225 250

The sample average working time is 252 minutes. The engineers believe that the known population standard deviation of 28 minutes also applies to the new power saver laptop.

Perform a hypothesis test of the claim that the true average working time for all batteries in power saver laptops is longer than 240 minutes. Use a 5% level of significance.

**Step 1: Hypotheses:**

Ho: The true average working time for all batteries in power saver laptops is 240 minutes (or less)

Ha: The true average working time for for all batteries in power saver laptops is longer than 240 minutes.

**Parameter:  $\mu$  = true average working time for batteries in all power saver laptops**

**Random Variable:  $\bar{X}$  = average working time for a sample of batteries in power saver laptops**

**Ho:  $\mu \leq 240$  (or  $\mu = 240$  is acceptable)**

**Ha:  $\mu > 240$  one tailed test to the right**

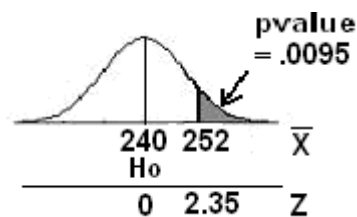
**Significance Level  $\alpha = 0.05$**

**Step 2:** For our sample of 30 laptops: average working time for batteries until recharging is  $\bar{x}=252$  minutes.

**Use a normal distribution  $N(240, 28/\sqrt{30})$  ; Z-Test** because  $\sigma = 28$  minutes is known.

**Step 3:** Find the test statistic and the p-value; interpretation of p-value, graph

**Use Calculator Ztest: test statistic is  $z = 2.35$  p-value is .0095**



**Ztest calculations:**

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{252 - 240}{28/\sqrt{30}} = 2.35$$

$$p\text{-value} = P(\bar{x} > 252 | \mu = 240) =$$

$$\text{normalcdf}(252, 10^{99}, 240, 28/\sqrt{30}) = .0095$$

*You should be able to this calculation either using the Z-test or by using the normalcdf*

**Interpreting the p-value :**

**If the population average is really  $\mu = 240$  minutes, the probability is .0095 of getting a sample average  $\bar{X}$  of 252 minutes or longer.**

**Step 4: Decision: REJECT the null hypothesis** because **p-value <  $\alpha$  (.0095 < .05)**

*Reasoning underlying the decision:*

*pvalue = .0095 is small. Getting this sample would be a rare event if Ho is true.*

*Therefore we do NOT believe that the assumption in Ho is true.*

**Step 5: Conclusion stated in the context of the problem:**

**At a 5% level of significance, the sample data show strong enough evidence to conclude that the true population average working time for all batteries in powersaver laptops is longer than 240 minutes.**

*NOTE: Find a confidence interval to estimate of the true average working time for batteries in power saver laptops, so that the marketing department will know what claims they can make on packaging and in advertising.*

$Z_{.025} = 1.96$  ;  $\bar{x} \pm EBM = \bar{x} \pm Z \sigma / \sqrt{n} = 252 \pm 1.96(28/\sqrt{30})$  We estimate with 95% confidence that the true average working time before recharging for all batteries in powersaver laptops is between 242 and 262 minutes.

**EXAMPLE F: Hypothesis Test of Population Mean  $\mu$  when Population Standard Deviation  $\sigma$  is NOT KNOWN**

At Dina's Dress shop, customers complain that fashion designers assume that all women are tall when they design clothes. Dina read that the designers whose clothes she carries tend to design for women who are 5 feet 6 inches (66 inches) tall. Because of her customers comments, Dina wants to determine whether the average height of all her customers is shorter.

A random sample of 25 customers has an average height of 64 inches with standard deviation 2.7 inches. (Assume that the population of individual customer heights is approximately normally distributed.) Perform a hypothesis test to determine if the average height of all Dina's customers is less than 66 inches. Use a 2% level of significance

**Step 1: Hypotheses:** Ho: The true average height of all customers at Dina's Dress shop is 66 inches.  
Ha: The true average height of all customers at Dina's Dress shop is less than 66 inches.

**Parameter:**  $\mu$  = true population average height of all customers at Dina's Dress shop

**Random Variable:**  $\bar{X}$  = average height of customers in a sample

**Ho:**  $\mu \geq 66$  (or  $\mu = 66$  is also acceptable)

**Ha:**  $\mu < 66$

**one tailed test to the left**

**Significance Level  $\alpha = 0.02$**

**Step 2:** We do NOT KNOW the population standard deviation  $\sigma$  for heights of all customers.

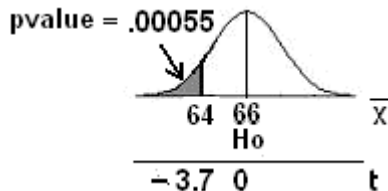
For our sample of  $n = 25$  customers: sample average height  $\bar{x} = 64$  inches; sample standard deviation  $s = 2.7$

**Use t-distribution with degrees of freedom  $df = n - 1 = 25 - 1 = 24$  ; Ttest because  $\sigma$  is not known**

*Technical Note: the problem says to assume that the population of individual customers heights is approximately normally distributed; this assumption of normal distribution for individual data allows us to use the t-distribution to test the average. If individual heights were significantly non-normally distributed, the t-test would not be appropriate and another test beyond the scope of Math 10 would be needed.*

**Step 3:** Find the test statistic and the p-value; interpretation of p-value, graph

**Use Calculator Ttest test statistic is  $t = -3.7$  p-value is .00055**



**Ttest calculations:**

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{64 - 66}{2.7/\sqrt{25}} = -3.7037$$

$$p\text{-value} = tcdf(-10^{99}, -3.7037, 24) = 5.5499E - 4$$

**Interpreting the p-value :**

If the average height of all customers at Dina's shop is really  $\mu = 66$  inches, the probability is .00055 of getting a sample average height  $\bar{x}$  of 64 inches or less

**Step 4: Decision: REJECT the null hypothesis** because **p-value  $< \alpha$  (.00055  $< .02$ )** .

*Reasoning underlying the decision:*

*pvalue = .00055 is small. Getting this sample would be a rare event if Ho is true.*

*Therefore we do NOT believe that the assumption in Ho is true.*

**Step 5: Conclusion stated in the context of the problem:**

**At a 2% level of significance, the sample data show strong enough evidence to conclude that the true population average height of all customers at Dina's Dress shop is less than 66 inches.**

*Note: Then find a confidence interval to estimate the true average height of all Dina's customers, so that she will be able to decide which designers to buy dresses from for the shop.*

With  $df = 24$ ,  $t_{.01} = -2.49$   $\bar{x} \pm EBM = \bar{x} \pm t s/\sqrt{n} = 64 \pm 2.49(2.7/\sqrt{25})$  We estimate with 98% confidence that the true average height of all Dina's customers is between 62.65 and 65.35 inches.

Source for height data: National Health and Nutrition Examination Survey (NHANES III), 1988–1994 and <http://www.biostat.jhsph.edu/bstcourse/bio751/papers/bimodalHeight.pdf>

**EXAMPLE G: Hypothesis Test of Population Proportion p**

A city government needs to determine whether the proportion (percent) of city residents without health insurance is different than the percent nationally. Nationally, it is believed that 16% of US residents lack health insurance. In a survey of 1600 city residents, 244 residents lack health insurance. Perform a hypothesis test using a 5% level of significance

**Step 1: Hypotheses:**

Ho: 16% of all city residents lack health insurance (same as national percent)

Ha: The true proportion of all city residents without health insurance is different from 16%

**Parameter: p = true population proportion of city residents without health insurance**

**Random Variable: P' = sample proportion of city residents without health insurance**

**Ho: p = .16**

**Ha: p ≠ .16                      two tailed test**

**Significance Level α = 0.05**

**Step 2:** This is a test of a proportion: In this sample of 1600 residents, 244 don't have health insurance

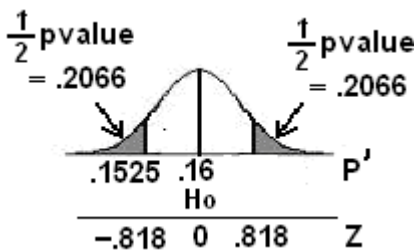
**Sample Proportion p' =  $\hat{p}$  = 244/1600 = 0.1525** (use either symbol  $\hat{p}$  or p')

**Use a normal distribution  $N(p, \sqrt{\frac{pq}{n}})$  with a 1-PropZTest**

*Technical Note: The conditions  $np > 5$  and  $nq > 5$  must be satisfied to use a 1 proportion Z test.. This ensures that the sample size is "large enough" to use this method.  
 $np = 1600(.16) = 256 > 5$  and  $n(1-p) = 1600(1-.16) = 1600(.84) = 1344 > 5$   
 Otherwise statisticians would use other methods not covered in Math 10.*

**Step 3:** Find the test statistic and the p-value; interpretation of p-value, graph

**Use Calculator 1-Prop Z Test: test statistic is z = -.818                      p-value is .4132**



**1-PropZTest calculations:**

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{.1525 - .16}{\sqrt{\frac{(.16)(.84)}{1600}}} = \frac{-.1525 - .16}{.009165} = -.818$$

$$p\text{-value} = \text{normalcdf}(0, .1525, .16, .009165) + \text{normalcdf}(.1675, 1, .16, .009165)$$

$$p\text{-value} = .2066 + .2066 = .4132$$

**Interpreting the p-value :**

**If the true population proportion is p = .16, then the probability is .4132 of getting a sample proportion p' ( $\hat{p}$ ) of .1525 or more extreme**

*(Note: "more extreme" means "further away")*

**Step 4: Decision: DO NOT REJECT the null hypothesis because p-value > α**  
**.4132 > .05**

Reasoning underlying the decision:

*Getting this sample if Ho is true would NOT be a rare event; the probability is .4132  
 The sample data do not contradict the null hypothesis..*

**Step 5: Conclusion stated in the context of the problem:**

**At a 5% level of significance, the sample data do not show strong enough evidence to conclude that true proportion of all city residents without health insurance differs from 16%. We continue to believe the null hypothesis that 16% of all city residents lack health insurance.**

**TRY IT:** It is believed that 20% of all residents in that state lack health insurance. Can we conclude that the true proportion (percent) of city residents without health insurance is different than the statewide percent?

**EXAMPLE H: Hypothesis Test of Population Proportion p**

The owners of Ace Auto Repair believe that at most half of all their customers drive foreign cars. One of their mechanics is taking Math 10 and as his student project performs a hypothesis test. He selects a random sample of 50 cars brought in for repair and find that 21 of them are foreign cars.

At a 5% level of significance, perform the hypothesis test.

**Step 1: Hypotheses:**

**Parameter:**  $p$  = true proportion of all cars repaired at Ace Auto Repair that are foreign cars  
**Random Variable:**  $P'$  = proportion of a sample of cars repaired at Ace Auto Repair in that are foreign cars

**Ho:**  $p \leq 0.50$       **Ha:**  $p > 0.50$       **one tailed test (right tail)**

**Significance Level  $\alpha = 0.05$**

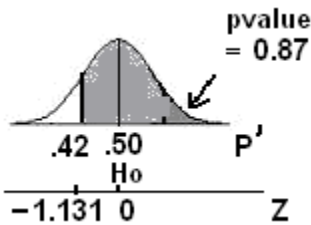
**Step 2:** This is a test of a proportion: In this sample of 50 cars, 21 are foreign.

**Sample Proportion**  $p' = \hat{p} = 21/50 = 0.42$  (use either symbol  $\hat{p}$  or  $p'$ )

Use a normal distribution  $N(p, \sqrt{\frac{pq}{n}})$  with a **1-PropZTest**

**Step 3:** Find the test statistic and the p-value; interpretation of p-value, graph

**Use Calculator 1-Prop Z Test: test statistic is  $z = -1.131$  p-value is .87**



**1-PropZTest calculations:**

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{.42 - .50}{\sqrt{\frac{(.50)(.50)}{50}}} = \frac{-.08}{.07071} = -1.131$$

$$p\text{-value} = \text{normalcdf}(0, .42, .5, .07071) = 0.871$$

**Interpreting the p-value :**

If the true population proportion is  $p = .42$ , then the probability is .87 of getting a sample proportion  $p'$  ( $\hat{p}$ ) of .50 or higher

**Step 4: Decision:** **DO NOT REJECT the null hypothesis** because  **$p\text{-value} > \alpha$**   
**.87 > .05**

Reasoning underlying the decision:

*Getting this sample if  $H_0$  is true would NOT be a rare event; the probability is .87  
The sample data do not contradict the null hypothesis..*

**Step 5: Conclusion stated in the context of the problem:**

At a 5% level of significance, the sample data do not show strong enough evidence to conclude that more than half of all customers at Ace Auto Repair own foreign cars.

We continue to believe the owners' assumption that at most half of all customers at Ace Auto Repair own foreign cars.