Hypothesis Test
Chapter 9

A Step by Step Guide

If using a printed handout of these slides,
the slides should be read left to right,
all of top row first, then all of bottom row.

If viewing as a slide show, keep clicking your mouse or pressing enter to reveal each slide, step by step, and to move to the next slide. On many slides it will take several clicks to see the entire slide.

Step 1 Hypotheses

- In the real world, decided before collecting data
- In a Math 10 problem, READ carefully.
- Reading skills are very important here!

- Look for words indicating $=$, $\geq$, or $\leq$
  to form the Null Hypothesis: Ho

- Look for words indicating $\neq$, $<$ or $>$
  to form the Alternate Hypothesis: Ha

Null Hypothesis: Ho
always contains equality of some type

= “is”, “equals”, “the same as”,
“no different than”, “has not changed”

≥ “greater than or equal to”, “at least”
“not less than”

≤ “less than or equal to”, “at most”
“no more than”, “ does not exceed”

Alternate Hypothesis: Ha always contains strict inequality

$\neq$ "does not equal", "not the same as"
"differs from", "different",
"has changed"

$>$ "greater than", "exceeds", "higher",
"larger", "bigger", "longer", "more"

$<$ "less than", "smaller", "lower",
"shorter", "under"

Significance Level α

- **Will always be small; is our "cutoff" between a large and small probability when making the decision.**
- In the real world, decided when determining hypotheses before collecting the data
- In Math 10, read the problem.
- If problem does not state the significance level use 5% (α =.05) in Math 10

Deciding What Test to Use

Mean, Population Standard Deviation Known:
ZTEST

Mean, Population Standard Deviation NOT Known:
TTEST

Proportion:
1PropZTEST

What Distribution?

Mean, Population Standard Deviation Known:

$$N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

Mean, Population Standard Deviation NOT Known:

t distribution

with $df = n - 1$

Proportion:

$$N\left(p, \sqrt{\frac{pq}{n}}\right)$$

Test Statistic

- t or z value shown on your calculator
- use the correct symbol
- tells us how many "appropriate standard deviations" the sample mean or proportion is away from the value in the null hypothesis

- The "appropriate standard deviation" is the standard deviation for a mean or proportion as appropriate for the problem: statisticians call this the "standard error"

p-value: Overview of the Theory

- The "p-value" is the probability of getting a sample that differs from the null hypothesis by as much as, or more than, your sample does, if the null hypothesis is true.

p-value: Overview of the Theory

- Suppose that the null hypothesis Ho is true:
- Is your sample “close to” the null hypothesis Ho? Is your sample reasonably likely to have occurred?
- Is your sample “far from” the null hypothesis Ho? Is your sample very unlikely to have happened (a rare event)?

p-value: Overview of the Theory

- Suppose that the null hypothesis Ho is true:
- Is your sample “close to” the null hypothesis Ho? Is your sample reasonably likely to have occurred?
- Then we have no reason to doubt the truth of the null hypothesis. We continue to believe that Ho might possibly be true.
- Decision :

  Do NOT REJECT the NULL HYPOTHESIS Ho

p-value: Overview of the Theory

- Suppose that the null hypothesis Ho is true:
- Is your sample "far from" the null hypothesis Ho? Is your sample very unlikely to have happened (a rare event)?
- The sample is REAL; the null hypothesis is a theory. An unlikely sample leads us to decide that the null hypothesis is probably not true
- Decision:
  REJECT the NULL HYPOTHESIS Ho.

Decision Rule

- If pvalue $\geq$ significance level α
  Decision: Do NOT Reject Ho

- If pvalue $<$ significance level α
  Decision: Reject Ho

Conclusion

- The conclusion is a sentence in the context of the problem that states
  - whether the data **DID** or **DID NOT** show strong enough evidence
  - to conclude that the alternate hypothesis is true.
- CONTEXT means
  - write out the alternate hypothesis as described in the words of the problem
  - don't say the "alternate hypothesis is true"

Conclusion

- If you Rejected Ho:
- Write a sentence that
  - Refers to the significance level
  - States in words in the context of the problem that you HAVE strong enough evidence to conclude that the alternate hypothesis is true.
  - REMEMBER CONTEXT!

Conclusion

- If you did NOT REJECT Ho:
- Write a sentence that:
  - Refers to the significance level
  - States in words in the context of the problem that you DO NOT HAVE strong enough evidence to conclude that the alternate hypothesis is true.
  - REMEMBER CONTEXT!

Example of Conclusion:

- X= cost of one statistics textbook
  μ = true population average cost for all statistics textbooks
- $\alpha = 0.05$
- Ho: $\mu = 100$ Ha: $\mu > 100$
- Suppose Decision is: REJECT Ho
- Conclusion: At a 5% level of significance the sample data provide strong enough evidence to conclude that the true average cost for all statistics textbooks is more than $100.

Example of Conclusion:

- X= cost of one statistics textbook
  μ = true population average cost for all statistics textbooks
- $\alpha = 0.02$
- Ho: $\mu = 100$ Ha: $\mu > 100$
- Suppose Decision is: DO NOT REJECT Ho
- Conclusion: At a 2% level of significance the sample data do NOT provide strong enough evidence to conclude that the true average cost for all statistics textbooks is more than $100

Why do we need a Conclusion?

- The DECISION to reject or not reject the null hypothesis needs to be translated back into a complete sentence stating the CONCLUSION in non-mathematical language, referring to the words of the problem, so that non-statisticians can understand the results.
- We include the significance level so that others reading it will know the criteria we used to reach our conclusion.

MORE CHAPTER 9 DETAILS

- The hypothesis test write-up, and even multiple choice exam questions, require to you understand the details of the calculation:
  - test statistic (letter symbol & numerical value)
  - pvalue
  - graph illustrating the pvalue
  - sentence interpreting the pvalue

Don’t ignore learning these details – you will lose many points if you don’t learn this! Remember it will take practice to get good at this.

(hence: homework!)

Drawing the Graph

Starting to draw the graph:
Both One-Tailed and Two-Tailed tests:

- Draw the shape representing the distribution and label the horizontal axis xbar or p'
- Under center of graph, below horizontal axis : mark and label the number cited in the NULL hypothesis Ho Write “Ho” and write the number under the center of the horizontal axis.
- Find the sample mean xbar or sample proportion p' ; locate it on the graph in relation to the center. Make a mark on the horizontal axis and write the value of xbar or p' under the mark, below the horizontal axis.

Drawing the Graph

Completing the graph: One-Tailed tests

- Look at the direction indicated by the inequality in the ALTERNATE HYPOTHESIS Ha
- Starting at the sample value xbar or p', shade in the direction indicated by Ha
- If Ha contains <:
  shade left from the value of xbar or p'
- If Ha contains >:
  shade right from the value of xbar or p'
- ABOVE the shaded area on graph, write pvalue =, state its numerical value, and make an arrow pointing to the shaded area.

Drawing the Graph

Completing the graph: Two-Tailed tests

- ALTERNATE HYPOTHESIS Ha has ≠
- Look at the point you marked for the sample value xbar or p'. Now mark another point that is the same distance away but on the opposite side of the center.
- Shade out from both points, toward the tails, away from the center.
- You will see two shaded areas.

ABOVE each shaded area: Label each of them ½ pvalue = and write the numerical value of half of the pvalue

Sample Graphs

The next few slides show some completed graphs for means and proportions, and for one tailed and 2 tailed tests.

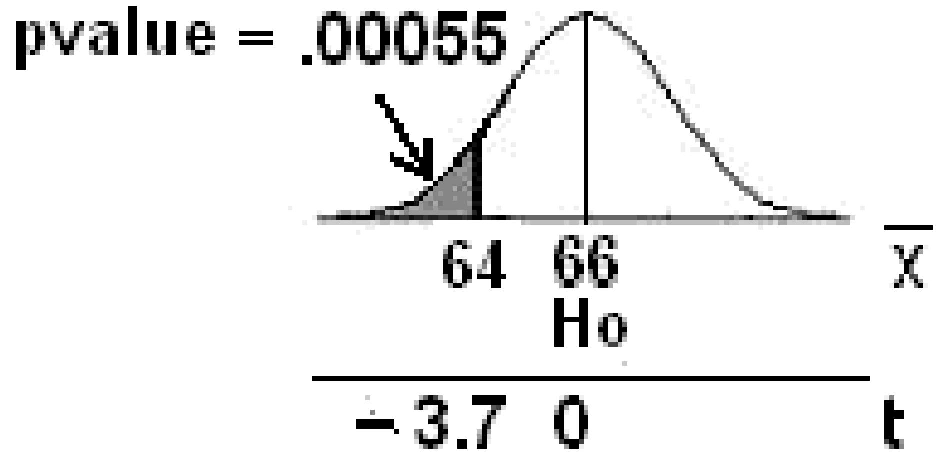
On some of the graphs, a second axis has been included to show the test statistic

(z or t), with 0 at center of the distribution for $Z \sim N(0,1)$ or for t.

Graph: Left Tailed (mean)

Ho: μ ≥ 66   Ha: μ< 66

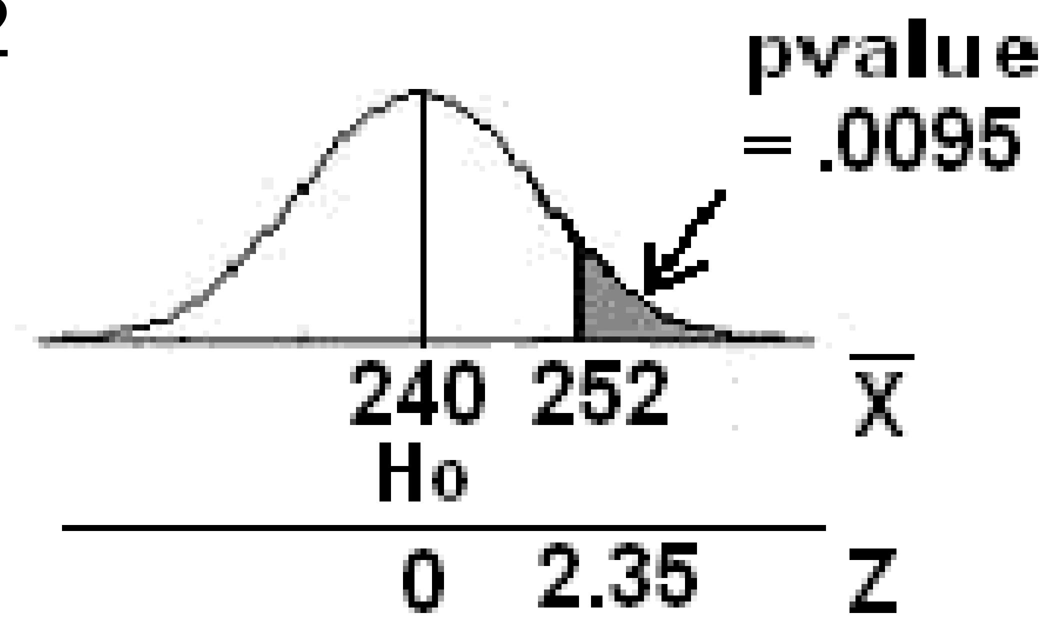
Xbar = 64



Graph: Right Tailed (mean)

Ho: μ ≤ 240   Ha: μ>240

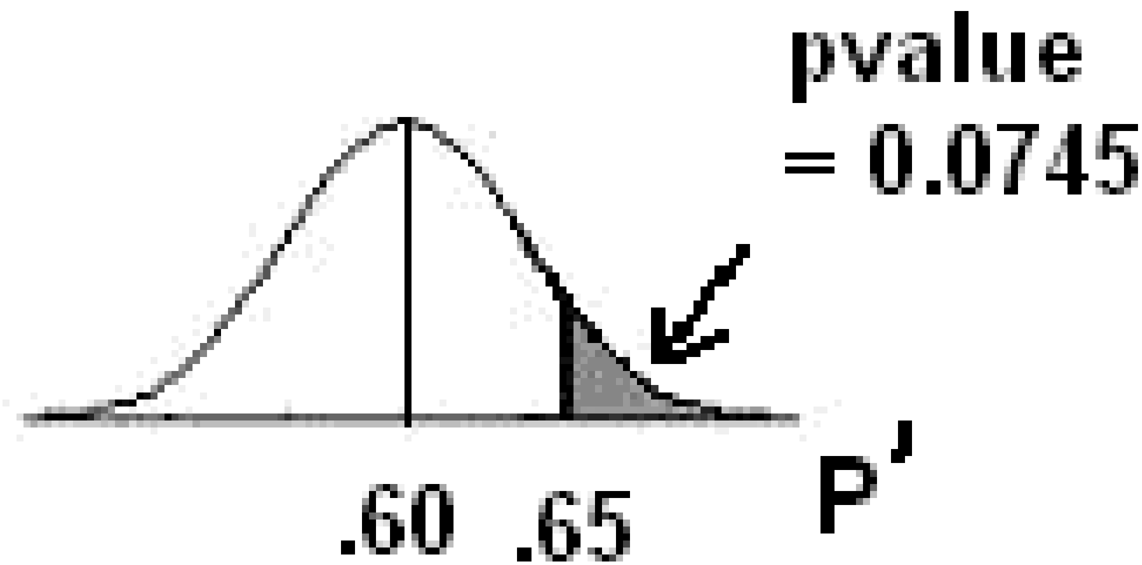
Xbar = 252



Graph: Right Tailed(proportion)

Ho: p ≤ .60   Ha: p> .60

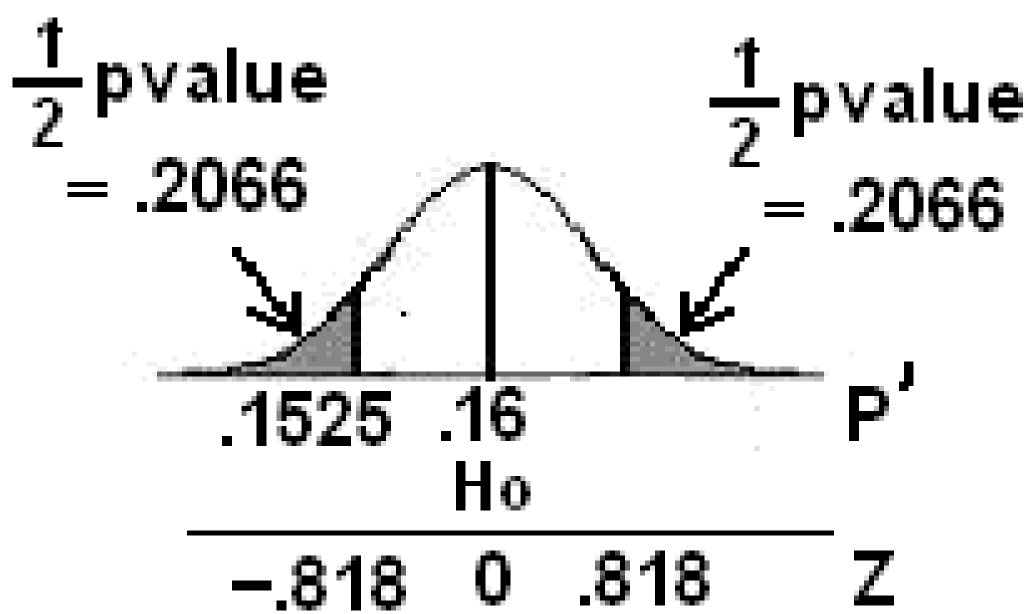
p' = .65



Graph: Two Tailed (proportion)

Ho: p = .16   Ha: p ≠ .16

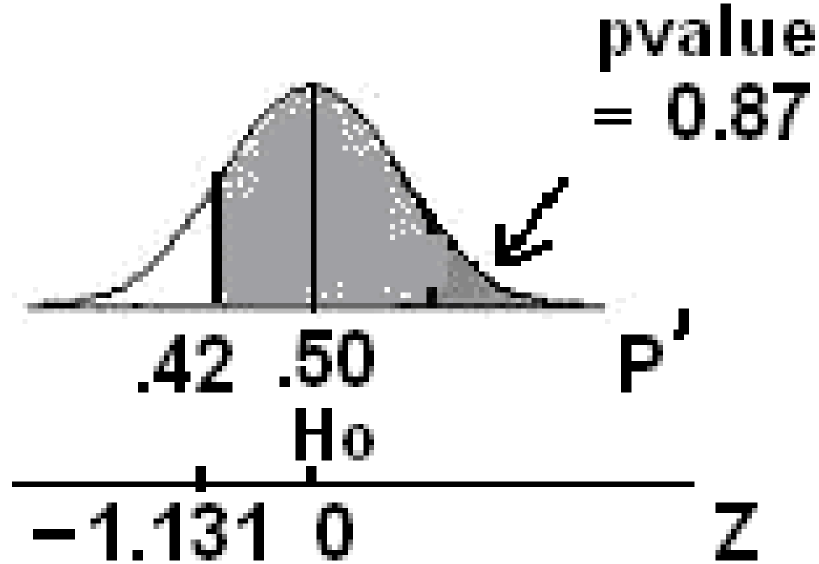
p' = .1525



Graph: Two Tailed (proportion)

Ho: p ≤ 0.50   Ha: p > 0.50

p' = 0.42



Interpreting the pvalue

- Interpretation has **3 parts**
- Describe null hypothesis (center of graph)
- State the p-value
- Describe shaded area (sample value and direction of alternate hypothesis Ha)

This will make more sense after the next few slides

Interpreting the pvalue: Mean

- If the null hypothesis true and μ = _____
  [*state value in Ho*]
- then the probability is _____
  [*state p-value*]
- of getting a sample mean xbar of ____
  [*state value of xbar*]
- or _______ [*choose a direction:* **more** *(if Ha has >),* **less** *(if Ha has <),* **more extreme** *(if Ha has ≠)*]

Interpreting the pvalue: Proportion

- If the null hypothesis true and p = _____

  [*state value in Ho*]

- then the probability is _____

  [*state p-value*]

- of getting a sample proportion p' of ____

  [*state value of p'*]

- or ________ [*choose a direction:* **more** *(if Ha has >),* **less** *(if Ha has <),* **more extreme** *(if Ha has ≠)*]

Example Interpreting pvalue:

- X= the weight of a melon at the market
  μ = true population average weight of all melons at the market (in pounds)
- Ho: $\mu \leq 4$ Ha: $\mu > 4$ $\alpha = 0.05$

- Suppose xbar is 4.5 and p-value is 0.03
- If the null hypothesis is true and $\mu = 4$, then there is a probability of 0.03 of getting a sample mean xbar of 4.5 or more.

Example Interpreting pvalue:

- X= the age of one child learning to ride a bicycle
  μ = true population average age for all children to learn to ride a bicycle
- Ho: $\mu = 8$ Ha: $\mu \neq 8$ $\alpha = 0.01$
- Suppose xbar is 7.2 and p-value is 0.16
- If the null hypothesis is true and $\mu = 8$, then there is a probability of 0.16 of getting a sample mean xbar of 7.2 or more extreme.
- Remember: "more extreme" means "further away"

Example Interpreting pvalue

- p = true population proportion of statistics students intending to transfer at the end of the current quarter.
- Ho: $p \geq 0.35$ Ha: $p < 0.35$ $\alpha = 0.02$
- Suppose p' = 0.28 and p-value is 0.04
- If the null hypothesis is true and p = 0.35 , then there is a probability of 0.04 of getting a sample proportion p' of 0.28 or less

Graph and Interpretation of pvalue
How are they related?

- The graph of the pvalue shows probability picture that explains the pvalue
- The interpretation of the pvalue explains in words what the graph shows
- The pvalue is a conditional probability : the null hypothesis is the condition (center of graph, “if” in the sentence)
- Pvalue = Probability of getting a sample at least as far from Ho as my sample, if Ho is true
- Pvalue = Probability of getting a sample at least as extreme as my sample | Ho is true