

Determining Hypotheses and Interpreting Type I and Type II Errors

Example:

Suppose that a hospital is testing a new surgery for a certain type of knee injury.

The hospital's surgery review board needs to decide whether to approve this type of surgery.

In general, many patients with this type of knee injury recover on their own without treatment, and surgery may have risks. Therefore, the surgery review board has decided that the hospital can perform this surgery as a clinical trial.

After much discussion of the medical considerations involved in this surgery, they decide to approve this type of surgery for future use if the clinical trial shows that the new surgery would cure more than 60% of all such injuries; otherwise they will not approve it.

p = true population proportion of all people with this knee injury who would be cured by this surgery

$H_0: p \leq .60$

$H_a: p > .60$ one tailed test to the right (right tailed test)

A Type I error would be to decide that this surgery cures more than 60% of all knee injuries of this type, when in reality it cures 60% or fewer of all such injuries.

A Type II error would be to decide that this surgery cures 60% or fewer of all knee injuries of this type, when in reality it cures more than 60% of all such injuries.

1. Engineering students at a college want to determine how the price of their calculus book at their bookstore compares to the price at other college bookstores. The calculus book costs \$150 at their bookstore. They will collect data about the price of this calculus book from 30 other bookstores nationwide and will use the sample data to decide whether the true average price of this book at all other colleges in the nation is different from the price of \$150 at their bookstore.

μ = true average cost of this calculus book for all college bookstores

$H_0: \mu = 150$

$H_a: \mu \neq 150$ two tailed test

A Type I error would be to decide that the true average cost for this book is different than \$150 when in reality the true average cost is \$150.

A Type II error would be to decide that the true average cost for this book is \$150 when in reality the true average cost is different than \$150.

2. A soda bottler wants to test whether 12 ounce soda cans filled at their plant are underfilled, on average, containing less than 12 ounces of soda.

μ = true average amount of soda in all cans filled at this bottling plant

$H_0: \mu \geq 12$ (or $\mu = 12$ is also acceptable)

$H_a: \mu < 12$

one tailed test to the left (left tailed test)

A Type I error would be to decide that the true average amount of soda in all cans is less than 12 ounces when in reality it is 12 ounces or more.

A Type II error would be to decide that the true average amount in all soda cans is 12 ounces or more when in reality it is less than 12 ounces.

3. Exercise Circuit, a fitness center, advertises a 30 minute workout that rotates clients exercising through all the fitness stations in 30 minutes. Clients who want longer workouts have complained, but other clients prefer the relatively quick 30 minute workout. The center conducts a survey to determine whether the average desired workout time for its clients is longer than the current 30 minute circuit.

μ = true average desired workout time for ALL clients

Ho: $\mu \leq 30$ (or $\mu = 30$ is also acceptable)

Ha: $\mu > 30$ one tailed test to the right (right tailed test)

A Type I error would be to decide that the average workout time desired by all clients is longer than 30 minutes when it really is 30 minutes or less.

A Type II error would be to decide that the average workout time desired by all clients is 30 minutes or less when it really is longer than 30 minutes.

4. It has been estimated that nationally, 16% of US residents do not have health insurance coverage. Suppose that a city wanted to determine whether the percent of city residents that do not have health insurance is different from the national average.

p = true proportion of all city residents who do not have health insurance

Ho: $p = 0.16$

Ha: $p \neq 0.16$ two tailed test

A Type I error would be to decide that the proportion of all city residents without health insurance is different from 16%, when in reality 16% of all city residents lack health insurance.

A Type II error would be to decide that 16% of all city residents lack health insurance when in reality the percent without health insurance is different than 16%.

5. The Center for Disease Control reports that only 14% of California adults smoke. A study is conducted to determine if the percent of De Anza college students who smoke is higher than that.

p = true proportion of all De Anza college students who smoke

Ho: $p \leq 0.14$ (or $p = 0.14$ is also acceptable)

Ha: $p > 0.14$ one tailed test (right tailed test)

A Type I error would be to decide that the more than 14% of all De Anza College students smoke, when in reality 14%, or less, smoke.

A Type II error would be to decide that 14%, or less, of all De Anza College students smoke, when in reality more than 14% smoke.

6. The average price of a cup of coffee in the airport is at least \$2.50

μ = the true average price for ALL cups of coffee sold in the airport

Ho: $\mu \geq 2.50$ (at least means \geq)

Ha: $\mu < 2.50$ (one tailed test to the left, because Ha has $<$)

A Type I error would be to decide that the true average price of all cups of coffee sold at the airport is less than \$2.50, when in reality the true average price is \$2.50 or more

A Type II error would be to decide that the true average price of all cups of coffee sold at the airport is \$2.50 or more, when in reality the true average price is less than \$2.50

7. The average amount of time that statistics students do statistics homework each night is 2 hours.

μ = the average time that ALL statistics students do statistics homework each night.

Ho: $\mu = 2$

Ha: $\mu \neq 2$ (two tailed test, because Ha has \neq)

A Type I error would be to decide that the true average time all statistics students do statistics homework is different than 2 hours, when in reality the true average time is 2 hours.

A Type II error would be to decide that the true average time all statistics students do statistics homework is 2 hours, when in reality the true average time is different than 2 hours.

8. At most half of all customers at Ace Auto Repair drive foreign cars.

p = the true proportion of ALL customers at Ace Auto Repair who drive foreign cars.

Ho: $p \leq 0.50$ (at most means \leq)

Ha: $p > 0.50$ (one tailed test to the right, because Ha has $>$)

A Type I error would be to decide more than 50% of all customers at Ace Auto Repair drive foreign cars, when in reality 50% or less drive foreign cars.

A Type II error would be to decide 50% or less of all customers at Ace Auto Repair drive foreign cars, when in reality more than 50% drive foreign cars.