9. Centripetal Acceleration

**Equipment list:**
circular motion apparatus  
pan or digital balance  
full weight set and hangers  
string (the string is stored by wrapping it around the "vertical pointer")  
stop watch  
meter stick

**Purpose:** In this lab you will calculate a centripetal acceleration and confirm the value of the force necessary to cause this acceleration.

**Theory:** Any body undergoing circular motion must be accelerating even if it is moving at constant speed since the direction of its velocity vector is continuously changing. Since acceleration is a vector it has a magnitude and a direction. It can be shown that the magnitude of the centripetal acceleration is $v^2/r$, where $r$ is the radius of the circle the body is moving in, and $v$ is the body's speed. The direction of the acceleration vector at any instant is always toward the center of the circle.

By Newton's 2nd Law, any body that is accelerating *necessarily* has a non-zero net force acting on it. In circular motion, the net force is often called the centripetal force, but it is important to understand that the centripetal force is the name of the net force acting on a body and always represents the vector sum of "real" forces acting on the body. The centripetal force is not itself a single, "real" force but the name given to the net force acting on a body causing it to move in a circle.

Leave at least one page in your theory section for further derivations.

**Introduction:** This lab is performed in two parts. In the first part a "bob" undergoes circular motion when spun by hand (see the diagram on the next page). By knowing the radius of the bob's circular path and the time it takes to complete one revolution, you can compute the magnitude of the centripetal acceleration of the bob, $a_c$.

In the second part of the lab, in a non-accelerating static situation, you will measure a force equivalent in magnitude (but not conceptually equivalent!) to the net force required to make the bob accelerate in the first part. This force is the weight of a hanging mass, $W_{hanging}$.

**The analysis consists of comparing $W_{hanging}$ to $M_{bob} a_c$. These two values should be equal.**

**Note:** The following paragraph is an in-depth explanation of why $W_{hanging}$ is equal to $M_{bob} a_c$.

To see that the magnitude of the static force measured in the second part of the lab is equal to the mass of the bob multiplied by the resultant acceleration of the bob, you must understand that the spring is stretched by the same amount in both the dynamic and static parts, and you must understand that if a spring is stretched by the same amount in each case, then the same magnitude of force is acting on it. However in the dynamic part of the lab (part 1), the spring exerts an inward force on the bob to keep the bob accelerating and this is *the only horizontal force acting on the bob*. By Newton's 3rd Law then, the bob exerts a force back on the spring outward thereby stretching the spring. In the second part of the lab (part 2), there are no
accelerations, the bob is in static equilibrium. The hanging weight pulls out on the bob via the connecting string (note: the tension in the connecting string is only equal to the hanging weight since there is no acceleration) and the spring pulls inward on the bob. Since the acceleration of the bob is zero, the net force acting on the bob is zero, so the force of the hanging weight on the bob is equal in magnitude to the force of the spring on the bob.

**Procedure:**

**Part 1. The dynamic part.**

**Preparation:** Setting the radius of the circle. With the spring not attached to the bob, allow the bob to freely hang without moving. Position the vertical pointer underneath the bob so the rod and bob match exactly. You now have set the radius of the circle. Measure the radius and record its value. You might have to move the horizontal position of the counter-weight and bob assembly in addition to the vertical pointer to achieve the correct alignment. Re-attach the spring to the bob. Use the leveling screws on the base of the apparatus to level the platform. You are now ready to "run" the dynamic part of the experiment.

**Running part 1.** Practice rotating the bob (while the spring is attached to it) by turning the vertical shaft. Make absolutely certain that the bob is swinging directly over the pointer at all times. If the bob is not directly over the pointer as it swings in its circle, then the string holding up the bob will have a horizontal component. The horizontal component of the string's tension force will alter the spring's force on the bob and your results will be inaccurate. As shown in the above diagram, position your hand near the bottom of the shaft where it is roughened for a good grip. You must rotate the shaft and therefore the bob at a constant speed. You can check to see if you are doing this because if you turn the shaft at the "correct" speed then the bob will be just over the vertical pointer. You may have to hold on to the bottom section of the whole apparatus to keep it from turning with the bob.

When you have gained confidence in turning the shaft at constant speed, have your lab partner use the counter-timer to time at least fifty total rotations. Call this time the total time, T_{total}. You now have the necessary data to calculate the centripetal acceleration of the bob.

**Record all your measured values in the data section of your lab book.**

In the theory section of your lab book (you *do* have the space, right?), derive an equation for the centripetal acceleration, a_c, in terms of the radius of the circle you measured and the total time of all fifty turns. **Your final equation should be a function of r and T_{total} only, no speed v!** Hint: the circumference of a circle is 2\( r \) and the speed of a body moving in a circle is 2\( r/t \), where \( t \) is time for one revolution. Use your final equation to find the numerical value of a_c in your calculation section.

**Part 2. The static part.**
Leave the spring attached to the bob. As shown in the diagram at the right, place an amount of mass on the hanger that stretches the spring so that the bob moves over the vertical pointer.

The value of the hanging mass's weight (include the hanger!) is equal in magnitude to the force that was required to make the bob accelerate in part 1. This equivalence is actually far from obvious but is worth some effort on the student's part to fully explain it.

**Part 3.** The last thing to do is to measure the mass of the bob. From Newton's 2nd Law, to equate the net force and the acceleration of a body, the mass of the body must be known. A question to answer is whether you should measure the mass of the spring as well as the bob.

You should repeat the above procedure for one or two more radii. Switch jobs with your partner. Consult your instructor for more details.

**Conclusion:** Compare your two values, $W_{\text{hanging}}$ and $M_{\text{bob}} a_c$, and discuss the results.

**Questions to consider.**
Speculate on possible sources of systematic errors that could be eliminated in future labs. In terms of systematic errors, would you expect $W_{\text{hanging}}$ or $M_{\text{bob}} a_c$ to be consistently larger? If you can think of a reason, explain why.
What is the experimental value of using fifty total rotations rather than just one or ten? What type of error does this technique minimize?

In the dynamic part of the lab, if the string tension force on the bob has a horizontal component pulling out on the bob, will this make the spring force on the bob larger than it should be or smaller than it should be? How would this change if the string tension force had a horizontal component pulling in on the bob? (Remember, there should be no horizontal component of the string tension force.)

**More things to do:**
For one configuration (i.e., one pointer position) the hanging mass weight, the radius, and the period are all related by one equation. With this equation in mind, take five data sets of those three parameters for five total configurations and construct a graph with the appropriate axis that gives a straight line whose slope can be related to the mass of the bob. Find the slope and calculate and compare the mass of the bob to its measured value found earlier.